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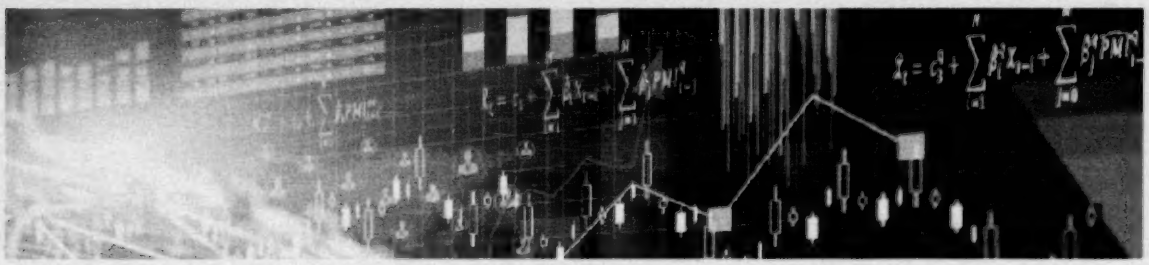
Monetary Commitment and the Level of Public Debt



by Stefano Gnocchi and Luisa Lambertini

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Monetary Commitment and the Level of Public Debt

by

Stefano Gnocchi[1] and Luisa Lambertini[2]

[1]Canadian Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
sgnocchi@bankofcanada.ca

[2]École Polytechnique Fédérale de Lausanne
luisa.lambertini@epfl.ch

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Abstract

We analyze the interaction between committed monetary policy and discretionary fiscal policy in a model with public debt, endogenous government expenditures, distortive taxation and nominal rigidities. Fiscal decisions lack commitment but are Markov-perfect. Monetary commitment to an interest rate path leads to a unique level of debt. This level of debt is positive if the central bank adopts closed-loop strategies that raise the real interest rate when inflation is above target owing to fiscal deviations. More aggressive defence of the inflation target implies lower debt and higher welfare. Simple Taylor-type interest rate rules achieve welfare levels similar to those generated by sophisticated closed-loop strategies.

JEL classification: E24, E32, E52
Bank classification: Credibility; Fiscal policy; Inflation targets; Monetary policy framework

Résumé

Nous analysons les interactions entre une politique monétaire à laquelle l'autorité responsable n'entend pas déroger et une politique budgétaire discrétionnaire, à l'aide d'un modèle caractérisé par l'existence d'une dette publique, des dépenses publiques endogènes, des distorsions fiscales et des rigidités nominales. Les décisions d'ordre budgétaire sont susceptibles d'être infléchies, mais elles sont parfaites au sens de Markov. L'engagement des autorités monétaires à s'en tenir à une trajectoire des taux d'intérêt donne lieu à un niveau de dette unique. Ce niveau de dette est positif si la banque centrale adopte des stratégies en boucle fermée qui lui font relever le taux d'intérêt réel lorsque, en raison de la décision de l'autorité budgétaire de s'éloigner de la politique budgétaire annoncée, l'inflation observée dépasse le taux cible. Une adhésion plus stricte au régime de ciblage de l'inflation se traduit par une baisse de la dette et un accroissement du bien-être. Un taux d'intérêt calculé selon de simples règles de Taylor permet d'atteindre des niveaux de bien-être comparables à ceux obtenus par l'application de stratégies complexes en boucle fermée.

Classification JEL : E24, E32, E52
Classification de la Banque : Crédibilité; Politique budgétaire; Cibles en matière d'inflation; Cadre de la politique monétaire

Non-Technical Summary

Monetary and fiscal authorities interact in several ways. For example, the policy rate affects the real value and financing cost of public debt, while government expenditures and taxes affect aggregate demand and supply and, ultimately, inflation. Also, most central banks commit to achieve mandated objectives that change only infrequently. In contrast, fiscal authorities revise their policies more often because of political turnover, which hampers any commitment to long-term fiscal plans. In this paper, we investigate the implications of monetary policy commitment for the optimal level of public debt.

We analyze the interaction between a monetary and a fiscal authority that are independent of each other and maximize social welfare in a model with monopolistic competition and nominal rigidities. The treasury finances government expenditures by levying distortionary income taxes and issuing public debt. The fiscal authority is allowed to change its plans at any time, while the central bank is assumed to commit once and for all to an interest rate rule that responds to deviations of inflation from its target. Both policy-makers face a trade-off between inflation and output. According to a conventional expectations-augmented Phillips curve, the promise to lower inflation in the future might be desirable for its expansionary effects. However, the promise is credible only if the fiscal authority has no incentive to revise its plan by generating surprise inflation.

We find that the optimal level of debt eliminates net gains from surprise inflation and is positive if the policy rate responds to inflation strongly enough to increase the real interest rate. The intuition is that, for a positive level of debt, inflation tightens the government's budget constraint through the monetary response: the higher refinancing cost of debt more than compensates for the lower real value of outstanding liabilities. A more aggressive defence of the inflation target increases the budgetary cost of surprise inflation and reduces the optimal level of public debt.

In light of their fiscal implications, our results give further support to inflation-targeting regimes. Such regimes contribute positively to the sustainability of fiscal plans and restrain the accumulation of public debt, thereby improving welfare by reducing the need to rely on distortionary taxes to finance interest rate payments.

1. Introduction

Most central banks are independent from the treasury and, in some countries, they are explicitly forbidden to purchase government bonds on the primary market. Nevertheless, monetary and fiscal authorities interact in several ways. As emphasized by Woodford (2001), monetary policy affects the real value and financing cost of outstanding public debt, while government spending and distortionary taxation affect inflation through their impact on aggregate demand and aggregate supply. Also, monetary and fiscal authorities differ in their ability to commit. Most central banks are accountable for achieving mandated objectives that change only infrequently. In contrast, fiscal authorities revise their policies more often because of political turnover, which hampers any commitment to long-term fiscal plans. In this paper, we study whether monetary policy commitment can reduce the costs of time inconsistency stemming from fiscal discretion and its implications for the optimal level of public debt.

We analyze monetary and fiscal policy interaction between two benevolent and independent authorities that maximize social welfare in a model with monopolistic competition and nominal rigidities. The treasury decides on government expenditures and distortionary income taxes, and can issue risk-free nominal bonds. While the fiscal authority is allowed to change its plans at any time, the central bank commits once and for all to an interest rate rule. Our most important result is that monetary policy affects the steady-state level of public debt and, via this mechanism, has first-order welfare effects.

We consider two types of monetary policy strategies. In the case of open-loop strategies, the central bank anticipates fiscal behaviour and optimally commits to an interest rate path that depends on exogenous shocks. Should the fiscal policy-maker deviate from its equilibrium strategy, the central bank sticks to its plan and tolerates the inflation fluctuations generated by fiscal deviations. In the case of closed-loop strategies, the central bank announces targets for the nominal interest rate and inflation.[1] The nominal interest rate is set equal to its target only if fiscal policy is consistent with the inflation target. Otherwise, the central bank stands ready to adjust the nominal interest rate enough to vary the real rate in response to inflation deviations from its target. We take the size of the interest rate response as given by the institutional environment and as representative of the central bank's aggressiveness in defending its inflation target.

We find that the steady-state level of public debt is negative under open-loop strategies and positive under closed-loop strategies. In the latter case, the more aggressive the central bank is in defending its inflation target, the lower the level of debt and, since taxes are distortionary, welfare will be higher. In our model, output is inefficiently low because of monopolistic competition and tax distortions so that both policy-makers face

[1] In practice, central banks might accept temporary deviations of inflation from their constant target because of shocks that generate a policy trade-off. We capture this flexibility by allowing our targets to vary over time.

a trade-off between inflation and output. According to a conventional expectations-augmented Phillips curve, the promise to lower inflation in the future might be desirable for its expansionary effects. However, the promise is credible if the fiscal authority has no incentive to revise its plan by generating surprise inflation, much along the lines of Kydland and Prescott (1977) and Barro and Gordon (1983). Accordingly, the fiscal authority rationally anticipates its future behaviour, taking into account how its future incentives are affected by current policies. The resulting optimal level of public debt is such that gains from surprise inflation are offset by its costs. Costs in turn relate to the budgetary effects of inflation, which crucially depend on monetary policy – namely, on the response of the real interest rate to deviations of inflation from its target. In the case of open-loop strategies the nominal interest rate does not respond to fiscal deviations, and inflation is costly because it reduces the real value of outstanding net public assets. Hence, it is optimal to accumulate assets up to the point that net gains from surprise inflation vanish. In contrast, in the case of closed-loop strategies, accumulating some debt is optimal. In fact, after a fiscal deviation the real interest rate increases and the higher cost of issuing new debt more than compensates for the fall in the real value of outstanding liabilities. The budgetary cost of inflation is higher the larger the interest rate response. As a result, a more aggressive defence of the inflation target reduces the optimal level of public debt.

Inspired by concrete practices, we also consider simple Taylor-type rules that specify a constant inflation target. This class of rules approximates closed-loop strategies relatively well in terms of welfare. This is because they also imply off-equilibrium threats that discourage fiscal deviations and thus favour low levels of steady-state debt. However, the implied responses to shocks are strongly suboptimal. Hence, the relatively good performance of these rules stems from their first-order effect on public debt rather than from their poor stabilization properties.

Our results do not question the separation of monetary and fiscal policy. Rather, they emphasize the important consequences of monetary-fiscal interactions. Fiscal discretion leads to suboptimal stabilization and excessive inflation volatility relative to fiscal commitment. However, a commitment to defend the inflation target promotes an environment with lower debt and higher welfare by mitigating the time-inconsistency problems of discretionary fiscal policy-making. Our results also speak to the importance of implementing flexible inflation targets, which improve the stabilization properties of policies. In addition to their well-known advantages relative to simple rules, they reduce the level of debt required to eliminate the government's incentives to rely on surprise inflation.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the model and Section 4 solves for the benchmark cases of efficiency and perfect coordination. We describe the policy strategies for the non-cooperative case in Section 5, and Section 6 presents our results. Section 7 compares the monetary regimes from the welfare point of view and Section 8 concludes.

2. Relevant Literature

A large strand of the literature investigates optimal monetary and fiscal policy under the assumption that a single authority chooses all policy instruments. In a real economy with exogenous government spending, flexible prices, state-contingent bonds and fiscal policy commitment, Lucas and Stokey (1983) show that the income tax and public debt inherit the dynamic properties of the exogenous stochastic disturbances. Chari et al. (1991) extend the Lucas and Stokey (1983) model to a monetary economy where the government issues nominal non-state-contingent debt. Optimal fiscal policy implies that the tax rate on labour remains essentially constant. On the other hand, inflation is volatile enough to make nominal debt state-contingent in real terms. Schmitt-Grohé and Uribe (2004a) build on Chari et al. (1991) by adding imperfectly competitive goods markets and sticky product prices. They find that a very small degree of nominal rigidity implies that the optimal volatility of inflation is low, while real variables display near-unit-root behaviour. Lack of stationarity arises because it is optimal to stabilize inflation, which makes it impossible to render debt state-contingent in real terms. This result parallels the contribution by Aiyagari et al. (2002), who find that the stationarity of real variables impinges on the ability of the government to issue state-contingent real debt. In particular, the key feature in determining the dynamic behaviour of tax rates and debt is whether markets are complete or can be completed by changing prices. All these contributions share the feature of considering a unique authority acting under commitment.

Recent work has shifted the focus to discretionary policy-making, retaining the assumption of a single policy authority. Diaz-Gimenez et al. (2008) assume that both monetary and fiscal policy are discretionary and find that public debt is positive at the steady state if the intertemporal elasticity of substitution of consumption is higher than one and negative otherwise. Debortoli and Nunes (2012) show that the lack of fiscal commitment is consistent with zero public debt in a real economy. In our monetary economy, we assume unitary elasticity of the intertemporal substitution of consumption, and we nest the result by Debortoli and Nunes (2012) when prices are flexible and/or when all markets are perfectly competitive. Campbell and Wren-Lewis (2013) consider an economy like ours to evaluate the welfare consequences of shocks at the efficient steady state and find substantially larger welfare costs of discretion relative to commitment. In contrast to their approach, we do not allow for lump-sum subsidies as an instrument to remove the monopolistic distortion at the steady state.

The literature on monetary and fiscal policy interaction is rather scant and has typically assumed a rich game-theoretic environment in simple macroeconomic models. Dixit and Lambertini (2003) explicitly model monetary and fiscal policies as a non-cooperative game between two independent authorities. The central bank can commit, while the fiscal authority acts under discretion. Their central bank is not benevolent but conservative as in Rogoff (1985), and the model is static. Niemann (2011) takes

the discretion literature one step further[2] and shows that the steady-state level of debt can be positive in a monetary economy where all policy-makers act under discretion and are non-benevolent. In our paper, we blend all these strands of the literature, but we always retain monetary policy commitment and focus our discussion on the implications of different monetary strategies for public debt.

3. The Model

We follow Schmitt-Grohé and Uribe (2004a) and consider a New-Keynesian model with imperfectly competitive goods markets and sticky prices. A closed production economy is populated by a continuum of monopolistically competitive producers and an infinitely lived representative household deriving utility from consumption goods, government expenditures and leisure. Each firm produces a differentiated good by using as an input the labour services supplied by the household in a perfectly competitive labour market. The prices of consumption goods are assumed to be sticky à la Rotemberg (1982).[3] For notational simplicity, we do not include a market for private claims, since they would not be traded in equilibrium. However, as in Chari et al. (1991), we can always interpret the model as having a complete set of state-contingent private securities. In addition, the household can save by buying non-state-contingent government bonds.

There are two policy-makers. We assume that the monetary authority decides on the nominal interest rate as in the cashless limit economy described by Woodford (2003) and Galí (2008). The fiscal authority is responsible for choosing the level of government expenditures, levying distortive taxes on labour income and issuing one-period nominal non-state-contingent government debt. By no arbitrage, the interest rate on bonds has to equalize the monetary policy rate in equilibrium.[4] Finally, we assume that the central bank and the fiscal authority are fully independent, i.e., they do not act co-operatively and they do not share a budget constraint.

This section briefly describes our economy and defines competitive equilibria.

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[2]Adam and Billi (2010) consider independent monetary and fiscal authorities acting under discretion to analyze the desirability of making the central bank conservative to eliminate the steady-state inflation bias, but they abstract from government debt.

[3]In order to keep the state-space dimension tractable, we depart from Calvo (1983) pricing, which introduces price dispersion as an additional state variable. Since Schmitt-Grohé and Uribe (2004a), this is a widespread modelling choice in the literature when solving for optimal policy problems without resorting to the linear-quadratic approach.

[4]We abstract indeed from default on the part of both the fiscal authority and private agents, so that the only bond traded at equilibrium is risk-free. We focus on monetary strategies that ensure the determination of the price level independently of fiscal policy, unlike Leeper (1991) and Woodford (2001).

3.1. Households

The representative household has preferences defined over private consumption, C_t, public expenditure, G_t, and labour services, N_t, according to the following utility function:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\chi) \ln C_t + \chi \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right], \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, E_0 denotes expectations conditional on the information available at time 0, φ is the inverse elasticity of labour supply and χ measures the weight of public spending relative to private consumption. Also, as we show below, χ determines government spending as a share of GDP, computed at the non-stochastic steady state of the Pareto-efficient equilibrium. C_t is a CES aggregator of the quantity consumed, $C_t(j)$, of any of the infinitely many varieties of goods, $j \in [0,1]$, and it is defined as

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\eta-1}{\eta}} \, dj \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where $\eta > 1$ is the elasticity of substitution between varieties. In each period $t \geq 0$ and under all contingencies, the household faces the following budget constraint:

$$\int_0^1 P_t(j) C_t(j) \, dj + \frac{B_t}{1+i_t} = W_t N_t (1-\tau_t) + B_{t-1} + T_t, \tag{3}$$

where $P_t(j)$ stands for the price of variety j, $W_t N_t (1-\tau_t)$ is after-tax nominal labour income and T_t represents nominal profits rebated to the household by firms. The household can purchase nominal government debt B_t at the price $1/(1+i_t)$, where i_t is the nominal interest rate. The nominal debt B_t pays one unit in nominal terms in period $t+1$. To prevent Ponzi games, the following condition is assumed to hold at all dates and under all contingencies:

$$\lim_{T \to \infty} E_t \left\{ \prod_{k=0}^{T} (1+i_{t+k})^{-1} B_{t+T} \right\} \geq 0. \tag{4}$$

Given prices, policies and transfers, $\{P_t(j), W_t, i_t, G_t, \tau_t, T_t\}_{t \geq 0}$, and the initial condition B_{-1}, the household chooses the set of processes $\{C_t(j), C_t, N_t, B_t\}_{t \geq 0}$, so as to maximize (1) subject to (2)-(4). After defining the aggregate price level[5] as

$$P_t = \left[\int_0^1 P_t(j)^{1-\eta} dj \right]^{\frac{1}{1-\eta}}, \tag{5}$$

[5]The price index has the property that the minimum cost of a consumption bundle C_t is $P_t C_t$.

as well as real debt, $b_t \equiv B_t/P_t$, the real wage, $w_t \equiv W_t/P_t$, and the gross inflation rate, $\pi_t \equiv P_t/P_{t-1}$, optimality is characterized by the standard first-order conditions:

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\eta} C_t, \tag{6}$$

$$\beta E_t \left\{\frac{C_t(1+i_t)}{C_{t+1}\pi_{t+1}}\right\} = 1, \tag{7}$$

$$\frac{N_t^{\varphi} C_t}{1-\chi} = w_t(1-\tau_t), \tag{8}$$

together with transversality:

$$\lim_{T\to\infty} E_t \left\{\beta^{T+1} \frac{b_{t+T}}{C_{t+T+1}\pi_{t+T+1}}\right\} = 0. \tag{9}$$

Equation (8) shows that the labour income tax drives a wedge between the marginal rate of substitution between leisure and consumption and the real wage.

3.2. Firms

There are infinitely many firms indexed by j on the unit interval $[0,1]$, and each of them produces a differentiated variety of goods with a constant return to scale technology:

$$Y_t(j) = z_t N_t(j), \tag{10}$$

where productivity z_t is identical across firms and $N_t(j)$ denotes the quantity of labour hired by firm j in period t. Following Rotemberg (1982), we assume that firms face quadratic price-adjustment costs:

$$\frac{\gamma}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2, \tag{11}$$

expressed in the units of the consumption good defined in (2) and $\gamma \geq 0$. The benchmark of flexible prices can easily be recovered by setting the parameter $\gamma = 0$. The present value of current and future profits reads as

$$E_t \left\{\sum_{s=0}^{\infty} Q_{t,t+s}\left[P_{t+s}(j)Y_{t+s}(j) - W_{t+s}N_{t+s}(j) - P_{t+s}\frac{\gamma}{2}\left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - 1\right)^2\right]\right\}, \tag{12}$$

where $Q_{t,t+s}$ is the discount factor in period t for nominal profits s periods ahead. Assuming that firms discount at the same rate as households implies that

$$Q_{t,t+s} = \beta^s \frac{C_t}{C_{t+s}\pi_{t+s}}. \tag{13}$$

Each firm faces the following demand function:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\eta} Y_t^d, \tag{14}$$

where Y_t^d is aggregate demand and it is taken as given by any firm j. Firms choose processes $\{P_t(j), N_t(j), Y_t(j)\}_{t\geq 0}$ so as to maximize (12) subject to (10) and (14), taking as given aggregate prices and quantities $\{P_t, W_t, Y_t^d\}_{t\geq 0}$. Let the real marginal cost be denoted by $mc_t \equiv w_t/z_t$. Then, at a symmetric equilibrium, where $P_t(j) = P_t$ and $N_t(j) = N_t$ for all $j \in [0,1]$, profit maximization and the definition of the discount factor imply that

$$\pi_t(\pi_t - 1) = \beta E_t\left[\frac{C_t}{C_{t+1}}\pi_{t+1}(\pi_{t+1} - 1)\right] + \frac{\eta z_t N_t}{\gamma}\left(mc_t - \frac{\eta - 1}{\eta}\right). \tag{15}$$

Equation (15) is the standard Phillips curve, according to which current inflation depends positively on future inflation and current marginal cost.

3.3. Policy-makers

There are two benevolent policy-makers in the economy. The monetary authority is responsible for setting the nominal interest rate i_t. The fiscal authority provides the public good, G_t, that is obtained by buying quantities $G_t(j)$ for any $j \in [0,1]$ and aggregating them according to

$$G_t = \left[\int_0^1 G_t(j)^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}}, \tag{16}$$

so that total government expenditures in nominal terms is $P_t G_t$, and the public demand for any variety is

$$G_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\eta} G_t. \tag{17}$$

Expenditures are financed by levying a distortive labour income tax τ_t or by issuing one-period, risk-free, non-state-contingent nominal bonds B_t. Hence, the budget constraint of the government is

$$\frac{B_t}{1+i_t} + \tau_t W_t N_t = B_{t-1} + G_t P_t. \tag{18}$$

The central bank and the fiscal authority determine the sequence $\{i_t, G_t, \tau_t\}_{t\geq 0}$ that, at equilibrium prices, uniquely determines the sequence $\{B_t\}_{t\geq 0}$ via (18). For what follows, the government budget constraint can be rewritten in real terms,

$$\frac{b_t}{1+i_t} + \tau_t mc_t z_t N_t = \frac{b_{t-1}}{\pi_t} + G_t, \tag{19}$$

after substituting for w_t from the expression for the real marginal cost.

Table 1: Benchmark calibration

Description	Parameter	Value
Weight of G in utility	χ	0.15
Weight of C in utility	$1-\chi$	0.85
Elast. subst. goods	η	11
Price stickiness	γ	20
Serial corr. tech.	ρ_z	0
Discount factor	β	0.99
Frisch elasticity	φ^{-1}	1

3.4. Competitive equilibrium

At a symmetric equilibrium where $P_t(j) = P_t$ for all $j \in [0,1]$, $Y_t(j) = Y_t^d$, the feasibility constraint is

$$z_t N_t = C_t + G_t + \frac{\gamma}{2}(\pi_t - 1)^2, \tag{20}$$

and the aggregate production function is $Y_t = z_t N_t$. Productivity is stochastic and evolves according to the following process:

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z, \tag{21}$$

where ϵ^z is an i.i.d. shock and ρ_z is the autoregressive coefficient.

We define the notion of competitive equilibrium as in Barro (1979) and Lucas and Stokey (1983), where decisions of the private sector and policies are described by collections of rules mapping the history of exogenous events into outcomes, given the initial state. To simplify notation, we stack private decisions and policies into vectors $x_t = (C_t, N_t, b_t, mc_t, \pi_t)$ and $p_t = (i_t, G_t, \tau_t)$, respectively. Let $s^t = (z_0, ..., z_t)$ be the history of exogenous events. Given a particular history, s^t, and the endogenous state, b_{t-1}, $x_r(s^r|s^t, b_{t-1})$ and $p_r(s^r|s^t, b_{t-1})$ denote the rules describing current and future decisions for any possible history s^r, $r \geq t$, $t \geq 0$. Finally, we can define a continuation competitive equilibrium as a set of sequences[6] $\mathcal{A}_t = \{x_r, p_r\}_{r \geq t}$ satisfying equations (7)-(9), (15) and (19)-(20) for any s^r. A competitive equilibrium $\mathcal{A}$ is simply a continuation competitive equilibrium starting at s^0, given b_{-1}.

3.5. Parameterization

The deep parameters of the model are set according to Table 1. The weight χ in the utility function has been chosen to roughly match the U.S. post-war government

[6]To simplify notation, we suppress functional arguments.

spending-to-GDP ratio. We set the serial correlation of the technological shock equal to zero to help us understand the mechanisms at play. After substituting the aggregate production function $Y_t = z_t N_t$, the log-linearized Phillips curve (15) reads as follows:

$$\hat{\pi}_t = \frac{\pi - 1}{2\pi - 1}\beta(\hat{C}_t - E_t\hat{C}_{t+1}) + \beta E_t\hat{\pi}_{t+1} + \\ + \frac{\eta Y mc}{\gamma\pi(2\pi - 1)}\hat{mc}_t + \frac{\eta Y}{\gamma\pi(2\pi - 1)}\left[mc - \frac{\eta - 1}{\eta}\right]\hat{Y}_t, \tag{22}$$

where a circumflex denotes log deviations from the steady state, and variables without a time subscript denote steady-state values. The effect of variations in the marginal cost on current inflation depends on the parameters γ and η but also on steady-state output and inflation, where the former depends on the initial level of government debt. Around a zero net inflation steady state, equation (22) boils down to

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{(\eta - 1)Y}{\gamma}\hat{mc}_t, \tag{23}$$

taking the same form as in the Calvo model. Hence, we can establish a mapping between our parameterization and average price duration. We set parameter γ equal to 20 for our benchmark calibration, which implies a price duration of roughly two quarters.

4. Pareto Efficiency and Policy Coordination

We take as benchmarks Pareto efficiency and the case of perfect coordination, where a single authority chooses monetary and fiscal policy instruments under commitment. We refer to these benchmarks in Section 6 to discuss the effects of fiscal discretion under a variety of monetary regimes. The Pareto-efficient allocation solves the problem of maximizing utility (1) subject to equations (2), (10), (16) and the resource constraint $Y_t(j) = C_t(j) + G_t(j)$ for any j. It can be shown that Pareto efficiency requires $C_t(j) = C_t$, $Y_t(j) = Y_t$, $G_t(j) = G_t$ and $N_t(j) = N_t$. Moreover, the marginal rate of substitution between leisure and private consumption and between leisure and public consumption must be equal to the corresponding marginal rate of transformation. This implies that

$$z_t = N_t^{\varphi}\frac{C_t}{1 - \chi} = N_t^{\varphi}\frac{G_t}{\chi}. \tag{24}$$

The optimality conditions yield the efficient allocation:

$$\overline{N}_t = 1; \quad \overline{Y}_t = z_t; \quad \overline{C}_t = (1 - \chi)z_t; \quad \overline{G}_t = \chi z_t. \tag{25}$$

Under Pareto efficiency, hours worked are constant, while consumption, government expenditures and output move proportionally to productivity. At the non-stochastic steady state, where $z_t = 1$ for all t, hours worked and output are equal to 1, while private and public consumption are equal to 0.75 and 0.15, respectively.

For the case of policy coordination, we follow the classic Ramsey (1927) approach and we define the optimal policy as a state-contingent plan. We refer to this case as Full Ramsey (FR). We define an FR equilibrium as a competitive equilibrium $\mathcal{A}_0$ that maximizes U_0, given the initial condition b_{-1}. The Lagrangian and the first-order conditions associated with the FR problem are reported in Appendix A.

Our benchmark economy features two distortions: (i) imperfect competition in the goods market; and (ii) price-adjustment costs. After setting $z_t = 1$ for all t, we analyze the non-stochastic steady state of the Ramsey equilibrium and consider three steady-state levels of government debt.[7] The first steady state is the efficient equilibrium of our model. This is the allocation where a labour subsidy completely eliminates the monopolistic distortion stemming from imperfect competition in the goods market. Since lump-sum taxes do not exist in our model, the labour subsidy, as well as the provision of the public good, must be financed with interest receipts on government assets. This implies that, at the efficient steady state,

$$b^{eff} = \frac{1/\eta - \chi}{1 - \beta}, \quad \tau^{eff} = -\frac{1}{\eta - 1}.$$

The Lagrangian multipliers on the government budget constraint λ^s, on the Euler equation λ^b and on the Phillips curve λ^p are equal to zero at the efficient steady state. The values of the macroeconomic variables of interest are reported in the third column of Table 2. Public assets must be 24 times GDP for the interest income to be sufficiently high to finance subsidies and government spending. Since public assets are private liabilities in our model, the assumption of a commitment to repay on the part of private agents may appear unrealistic with such a high level of indebtedness. But we consider the efficient steady state as a theoretical benchmark and maintain the assumption that all debts are repaid – private or public. The second steady state features a positive level of government debt that, without loss of generality, we set equal to GDP. In the third steady state the government is a creditor and public assets are equal to GDP. These two steady states are summarized in the fourth and fifth column of Table 2. In the economy with positive public debt, the tax rate is 17.35% at the steady state. The economy with public credit (negative public debt) has a steady-state tax rate of 15.18%, which implies more hours and increased output, relative to the economy with positive public debt. However, in both cases, hours worked, consumption and output are well below their efficient levels. Even under perfect coordination of monetary and fiscal policy, the economy fluctuates around a distorted steady state, unless the government accumulates a large stock of public assets that may be regarded as implausibly high.

In Section 6, we examine the dynamics of macroeconomic variables conditional on

[7] More precisely, we choose the steady-state level of debt knowing that there exists an initial condition b_{-1} that supports it. We abstract from the transition from such an initial condition to the chosen steady state.

Table 2: Steady state

Variable	Symbol	Value		
		Efficient	$b/Y = 1$	$b/Y = -1$
Consumption	C	0.85	0.74	0.75
Government expenditure	G	0.15	0.13	0.13
Hours worked	N	1	0.87	0.88
Real debt	b	-24.0909	0.87	-0.88
Income tax	τ	-0.1	0.1735	0.1518
Gross inflation	π	1	1	1

Note: b/Y is the quarterly debt-to-GDP ratio.

technological shocks at the FR equilibrium and compare them with the dynamics under fiscal discretion.

5. The Interaction of Monetary Commitment and Fiscal Discretion

In this section, we model policy-making as a non-co-operative game where monetary and fiscal policies are conducted by two separate and independent authorities. We assume that both policy-makers are benevolent and maximize social welfare, but only the monetary authority can credibly commit to future policies. In contrast, the fiscal authority cannot do so and therefore acts under discretion. We are interested in analyzing time-consistent fiscal policy under a variety of monetary arrangements, within the class of monetary commitment. Hence, we first describe the game in a general form, defining timing and strategy space. We do so by following the same formalism as in Chari and Kehoe (1990) and Atkeson et al. (2010). Then, we consider alternative policy regimes by varying the restrictions we impose on monetary strategies and computing the resulting equilibrium.

5.1. The policy game

A formal description of the game allows us to be transparent about the assumptions we make about the strategies available to the monetary and fiscal authorities. We focus on the strategic interaction between policy-makers and regard households and firms as non-strategic. Hence, there are only two players: the central bank and the fiscal authority.

Timing – The events of the game unfold according to the following timeline. In period $t = 0$, at a stage that one may consider as constitutional, the central bank commits once and for all to a rule, say σ_m. Then, in every period $t \geq 0$, (i) shocks occur and are perfectly observed by all agents and authorities; (ii) the fiscal authority chooses its

fiscal tools; and (iii) the monetary authority implements the plan it committed to at the constitutional stage and economic variables realize. The vector $q_t \equiv (z_t, G_t, \tau_t, i_t, x_t)$ represents chronologically the events that occur in each period. Accordingly, the history of the game can be defined as $h_t \equiv (q_t, h_{t-1})$ for $t > 0$ and $h_0 \equiv (q_0, b_{-1})$ for $t = 0$. Our timing assumption implies that the central bank leads both the fiscal policy-maker and private agents, since it chooses its policy at the constitutional stage. The fiscal policy-maker only leads private agents within each period. However, as will become evident below, it is convenient to think of the fiscal policy-maker as a sequence of authorities with identical preferences, each one leading their successors.[8]

Histories, strategies and competitive equilibrium – The fiscal authority faces history $h_{t,f} \equiv (h_{t-1}, z_t)$, i.e., it chooses government spending and taxes after observing h_{t-1} and the shock, so that its strategy is $\sigma_f = \{G_t(h_{t,f}), \tau_t(h_{t,f})\}_{t\geq 0}$. Similarly, the monetary authority faces history $h_{t,m} \equiv (h_{t-1}, z_t, G_t, \tau_t)$ and chooses its instrument according to strategy $\sigma_m = \{i_t(h_{t,m})\}_{t\geq 0}$. For any strategy, it is convenient to define its continuation from a given history. For instance, consider fiscal strategy σ_f. We denote its continuation as $\sigma_f^t = \{G_r(h_{r,f}), \tau_r(h_{r,f})\}_{r\geq t}$. Starting from any history h_{t-1}, and given a sequence of exogenous events from period t onward, fiscal and monetary strategies generate policies denoted by $\{p_r\}_{r\geq t}$, as in Section 3.4. Given these policies, private agents face information $h_{t,x} \equiv (h_{t-1}, z_t, G_t, \tau_t, i_t)$ and take decisions according to $\sigma_x = \{x_r\}_{r\geq t}$, where x_r are the decision rules defined in Section 3.4. Hence, once monetary and fiscal policy strategies are set, they generate a continuation competitive equilibrium $\mathcal{A}_t = \{x_r, p_r\}_{r\geq t}$ from any history h_{t-1}.

Strategy restrictions – We restrict to fiscal Markov strategies where fiscal instruments respond only to the inherited level of debt, b_{t-1}, and the history of exogenous events, s^t:[9]

$$G_t = G_t(s^t, b_{t-1}); \qquad \tau_t = \tau_t(s^t, b_{t-1}). \tag{26}$$

We consider monetary strategies of the following form:

$$i_t\left(s^t, b_{-1}, \tau_t, G_t\right) = \left(1 + i_t^T\left(s^t, b_{-1}\right)\right)\left[\frac{\pi_t}{\pi_t^T\left(s^t, b_{-1}, \tau_t, G_t\right)}\right]^{\phi_\pi} - 1, \tag{27}$$

where i_t^T and π_t^T denote the central bank's targets for the nominal interest rate and inflation, respectively.[10] The targets and the elasticity of the interest rate response

[8] If one restricts to the case of monetary commitment, inverting the order of moves within each period would not change our results. In fact, the central bank could still condition the nominal interest rate on the history of fiscal instruments.

[9] When modelling discretion, it is standard to assume that the policy authority does not respond to past policies in order to exclude a multiplicity of reputational equilibria. Chari and Kehoe (1990), King et al. (2008) and Lu (2013) discuss the cases of trigger strategies and reputation mechanisms. We follow the literature and require differentiability of the fiscal strategies as in Klein et al. (2008), Debortoli and Nunes (2012) and Ellison and Rankin (2007).

[10] We omit functional arguments unless they are required to avoid ambiguities.

to inflation, ϕ_π, are predetermined at the constitutional stage; ϕ_π is a constant and we restrict it to guarantee that the system of equations (7)-(9), (15), (19)-(20) and (26)-(27) has a locally unique solution. In particular, we assume that $\phi_\pi > 1/\beta$ so that the Taylor principle holds. We regard ϕ_π as an institutional parameter and take it as given, in the same way as we assume that institutions are designed to enforce monetary commitment. Section 7 discusses the optimal choice of ϕ_π from the welfare perspective.

This particular class of monetary strategies is appealing for various reasons. First, any competitive equilibrium can be implemented by choosing σ_f and σ_m within the class defined by (26) and (27). For example, consider a competitive equilibrium $\bar{\mathcal{A}}$ and its continuations $\mathcal{A}_t$. Take $(i_t, \pi_t) \in \mathcal{A}$, $(G_t, \tau_t) \in \mathcal{A}_t$ and specify monetary and fiscal strategies as follows: $G_t = G_t$, $\tau_t = \tau_t$, $i_t^T = i_t$ and $\pi_t^T = \pi_t$. Then, $\mathcal{A}$ is the locally unique solution to equations (7)-(9), (15), (19)-(20) and (26)-(27).[11] In other words, for any competitive equilibrium, we can find a pair of rules, (26) and (27), that can support it: our assumption on strategies is not particularly restrictive and simplifies the solution of the game. Second, the monetary rule is flexible enough to accommodate all the monetary policy regimes that we describe in the following sections. Specifically, we consider the case of open-loop strategies in Section 5.2, where monetary policy does not respond to the actions of the fiscal authority. We then consider the case of closed-loop strategies in Sections 5.3 and 5.4, where the central bank conditions the interest rate to fiscal policy. In particular, the central bank threatens to vary the nominal interest rate if fiscal policy compromises the achievement of the inflation target, i.e., $\pi_t \neq \pi_t^T$. In accordance with the mandates of most inflation-targeting central banks, we assume that the interest rate rule does not directly target fiscal variables.

Markov-perfect fiscal policy – We always maintain the assumption that fiscal decisions are Markov-perfect. Intuitively, the current fiscal authority chooses its instruments G_t and τ_t, taking into account that future fiscal policies will also be chosen optimally. Formally, a fiscal strategy σ_f^* is Markov-perfect if it maximizes U_t for any $h_{t,f}$ and for any monetary strategy σ_m, given continuation $\sigma_f^{*,t+1}$. We adopt a primal approach and solve for the policy problem by deciding on both policy variables and private decisions, subject to the constraint that they must be a continuation competitive equilibrium. Hence, we look for a competitive equilibrium that solves the following problem:

$$W_t^f = \max_{C_t, N_t, b_t, mc_t, \pi_t, G_t, i_t} \left\{ (1-\chi) \ln C_t + \chi \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} + \beta E_t W_{t+1}^f \right\} \tag{28}$$

subject to[12]

$$z_t N_t - C_t - G_t - \frac{\gamma}{2}(\pi_t - 1)^2 = 0, \tag{29}$$

[11]The proof is reported in Appendix B.4.

[12]We have substituted for τ_t into (19) from the household's optimality condition (8).

$$\frac{1-\chi}{C_t(1+i_t)} - \beta E_t \frac{1-\chi}{C_{t+1}(s^{t+1},b_t)\Pi_{t+1}(s^{t+1},b_t)} = 0, \tag{30}$$

$$\frac{b_t}{1+i_t} + \left(mc_t z_t - \frac{N_t^{\varphi} C_t}{1-\chi}\right) N_t - \frac{b_{t-1}}{\pi_t} - G_t = 0, \tag{31}$$

$$\beta E_t \frac{C_t \Pi_{t+1}(s^{t+1},b_t)(\Pi_{t+1}(s^{t+1},b_t)-1)}{C_{t+1}(s^{t+1},b_t)} + \frac{\eta}{\gamma} z_t N_t \left(mc_t - \frac{\eta-1}{\eta}\right) - \pi_t(\pi_t - 1) = 0, \tag{32}$$

and equation (27), taking b_{t-1} and i_t^T as given and function π_t^T into account. Say that the competitive equilibrium $\bar{\mathcal{A}}$ solves problem (28) for any t. $C_{t+1}(s^{t+1},b_t)$ and $\Pi_{t+1}(s^{t+1},b_t)$ are functions that belong to the continuation $\bar{\mathcal{A}}_{t+1}$, i.e., they describe equilibrium private consumption and inflation at time $t+1$.[13] They are taken as given by the fiscal authority because it cannot commit to future outcomes. However, the current level of debt, b_t, affects future inflation and consumption, which in turn enter the fiscal decision problem via equations (30) and (32). We assume that the fiscal policy-maker internalizes this effect to guarantee Markov-perfection. Rules $G_t(s^t,b_{t-1})$ and $\tau_t(s^t,b_{t-1})$ are an equilibrium fiscal strategy if they belong to continuations $\mathcal{A}_t$ for any t.

5.2. Open-loop monetary policy strategies

We start by considering the case where monetary policy is optimal according to the classic Ramsey (1927) approach, which prescribes an interest rate path that depends only on the history of exogenous events and the initial level of debt. More precisely, we say that rule (27) is open-loop if the following property holds: $\pi_t = \pi_t^T$ for any τ_t and G_t. Function π_t^T and parameter ϕ_π are irrelevant for the fiscal authority, because $(\pi_t/\pi_t^T)^{\phi_\pi} = 1$ and the nominal interest rate, $i_t = i_t^T(s^t,b_{-1})$, is not affected by fiscal variables. Therefore, under open-loop strategies, internalizing the monetary rule is equivalent to taking the nominal interest rate as given in problem (28). We define an equilibrium in open-loop strategies as follows: (i) fiscal policy σ_f^* is Markov-perfect; and (ii) given σ_f^*, the optimal monetary strategy, σ_m^*, maximizes U_0 in the class of open-loop strategies of the form (27).

We solve for the equilibrium of the game by backward induction. First, we look for a competitive equilibrium that satisfies (i) and thus solves problem (28). Second,

[13] We omit functional arguments unless they are required to avoid ambiguities. Since C_{t+1} and Π_{t+1} describe equilibrium consumption and inflation, they are unknown at the time of solving problem (28). However, solving (28) requires that we know the derivative of functions C_{t+1} and Π_{t+1} with respect to b_t. This is a conventional fixed-point problem that arises with Markov-perfect equilibria and has been tackled by Klein et al. (2008). We solve the problem with a second-order perturbation method, which has proven to be accurate in similar cases (Azzimonti et al. (2009)). We provide further technical details in Appendix C.

a competitive equilibrium $\mathcal{A}^*$ satisfies (ii) and is optimal for the monetary authority if it maximizes U_0 subject to constraints (9), (29)-(32) *and* the first-order conditions of problem (28). Equilibrium fiscal strategies are chosen from $\mathcal{A}_t^*$. Finally, we construct the monetary strategy. After setting $i_t^T = i_t^* \in \mathcal{A}^*$, if the fiscal authority always plays the equilibrium strategy, we also choose $\pi_t^T = \pi_t^*$ from $\mathcal{A}^*$. Suppose instead that the fiscal authority deviates from equilibrium at t and plays $\tilde{G}_t$ and $\tilde{\tau}_t$, while it reverts to equilibrium from $t+1$. Then, $i_t^T = i_t^*$ and the fiscal instruments, together with equations (9), (29)-(32) and rules $\mathcal{A}_{t+1}^*$, determine variables $\tilde{x}_t$, including inflation. Then, choose $\pi_t^T = \tilde{\pi}_t$. Since the central bank adjusts its target after a deviation, $(\pi_t/\pi_t^T)^{\phi_\pi} = 1$ as initially assumed, irrespective of whether fiscal policy deviates from equilibrium or not. Notice that even if the monetary policy instrument does not respond to fiscal policy, the central bank fully internalizes fiscal behaviour: the optimality conditions of the fiscal policy problem (28) are taken into account in the monetary policy problem.

Thus, the realized interest rate and inflation coincide with their corresponding targets, which fully describe the Ramsey optimal monetary policy under fiscal discretion. Should fiscal policy threaten the achievement of the inflation target, an event that is never observed at equilibrium, the central bank sticks to the announced interest rate but it compromises on its inflation target: the central bank stands ready to accommodate fiscal "misbehaviour". The formal problem is presented in Appendix B.1.

5.3. Closed-loop monetary policy strategies

We now assume that the inflation target is a function of exogenous events and the initial level of debt, i.e., $\pi_t^T(s^t, b_{-1})$. As a result, given i_t^T and π_t^T, the nominal interest changes with the fiscal instruments: if fiscal policy implies a deviation of inflation from the target, the monetary authority varies the nominal interest rate with elasticity ϕ_π. We label this class of strategies as closed-loop. The coefficient ϕ_π describes the extent to which the central bank tolerates "disagreement" with the fiscal authority on inflation. Given the institutional set-up, we define the equilibrium as follows: (i) fiscal policy σ_f^* is Markov-perfect; and (ii) given σ_f^* and ϕ_π, the optimal monetary strategy, σ_m^*, maximizes U_0 in the class of closed-loop strategies of the form (27).

As in the previous section, we solve the game by backward induction. A competitive equilibrium $\mathcal{A}^*$ is optimal for the monetary authority if it maximizes U_0 subject to constraints (9), (29)-(32) and the first-order conditions of problem (28). Optimal strategies of the form (26) and (27) can be designed by choosing G_t^* and τ_t^* from continuations $\mathcal{A}_t^*$ and targets i_t^T and π_t^T from $\mathcal{A}^*$. If the fiscal authority generates inflation $\pi_t \neq \pi_t^T$, the nominal interest rate endogenously responds by ϕ_π, as we correctly assume in problem (28). This is achieved by committing to interest rate and inflation targets that do not depend on fiscal instruments.

As with open-loop strategies, the realized interest rate and inflation coincide with their corresponding targets if the fiscal authority does not deviate from equilibrium. However, if fiscal policy pushes inflation above the target, an event that is never observed in equilibrium, the monetary authority tightens the monetary policy stance.

This off-equilibrium response affects the behaviour of the fiscal authority, as we illustrate below by comparing open- and closed-loop strategies. The formal problem is presented in Appendix B.2.

5.4. Simple Taylor-type rules

Finally, we consider the case where the central bank commits once and for all to a constant inflation target of its choice:

$$\pi_t^T = \pi^*, \quad i_t^T = \frac{\pi^*}{\beta} - 1, \ \forall t. \tag{33}$$

Then, the nominal interest rate target i^T is also constant and equal to the steady-state value of the nominal interest rate consistent with the inflation target. This monetary strategy belongs to the class of rules suggested by Taylor (1993) and is often used in dynamic stochastic general-equilibrium models to describe the behaviour of inflation-targeting central banks. In this regime, neither the target nor the elasticity depend on current economic conditions or the history of the game. Hence, as in the case of the strategies considered in Section 5.3, the central bank commits to change the nominal interest rate in response to any deviation of current inflation relative to its target. The equilibrium in this class of monetary rules can be seen as a particular case of the one we find in Section 5.3 and is defined as follows: (i) fiscal policy σ_f^* is Markov-perfect; and (ii) the monetary strategy (27) satisfies restriction (33). The formal problem is presented in Appendix B.3.

6. Results

6.1. Steady state

The previous sections described three alternative strategic environments for monetary and fiscal policy. In this section, we describe and compare the steady states associated with these environments, putting emphasis on the level of debt. Table 3 shows the steady-state values of all the macroeconomic variables of interest under open-loop monetary strategies, closed-loop strategies with $\phi_\pi = 1.5$ and a simple Taylor-type rule with $\phi_\pi = 1.5$ and $\pi^* = 1$.

The first-order condition of the fiscal authority relative to inflation, evaluated at the steady state, implies that

$$-\frac{\lambda^s b}{\pi^2}(\beta\phi_\pi - 1) - \lambda^f\gamma(\pi - 1) - \lambda^p(2\pi - 1) - \beta\phi_\pi\frac{\lambda^b}{C\pi^2} = 0, \tag{34}$$

where $\lambda^s \geq 0$ is the Lagrangian multiplier for the government budget constraint, $\lambda^f \geq 0$

for the resource constraint, $\lambda^p \leq 0$ for the Phillips curve and λ^b for the Euler equation.[14] The first term captures the effect of inflation on the public accounts. An increase in current inflation reduces debt repayment but raises the nominal interest rate with elasticity ϕ_π. If $\phi_\pi > 1/\beta$ and the real interest rate also increases, inflation leads to a loss in real terms, and the government has an incentive to generate deflation if it is a debtor. If, instead, the central bank does not respond to the actions of fiscal policy, the impact of inflation on the real cost of debt is only given by $\lambda^s b/\pi^2$: positive debt gives the incentive to generate inflation, negative debt to reduce it. The second term is the resource cost, which disappears if net inflation is zero or if prices are fully flexible ($\gamma = 0$). The third term captures the benefits from reduced price markups. With sticky prices, an increase in trend inflation reduces the average markup and raises output, which is suboptimally low owing to monopolistic competition. The last term is the effect on consumption smoothing. Higher inflation leads to lower government bond prices. Households are thus willing to defer consumption and, *ceteris paribus*, public debt tends to increase. This effect raises or reduces welfare, depending on how debt accumulation affects the inflation-output trade-off. From the first-order condition relative to debt:

$$\frac{\lambda^b}{C\pi^2} = -\lambda^p \frac{\frac{\partial \Pi}{\partial b_t} C(2\pi - 1) - \frac{\partial \mathcal{C}}{\partial b_t}\pi(\pi - 1)}{\frac{\partial \Pi}{\partial b_t} C + \frac{\partial \mathcal{C}}{\partial b_t}\pi}. \tag{35}$$

Under our calibration, the denominator on the right-hand side of (35) is positive under all monetary policy regimes. If the numerator is also positive, debt accumulation increases the forward-looking component of the Phillips curve and worsens the inflation-output trade-off. As we discuss below, the monetary policy regime determines whether debt accumulation improves or worsens the inflation-output trade-off and thus the sign of $\lambda^b/C\pi^2$.

First-order conditions (34) and (35) imply that[15]

$$b = \frac{\gamma\pi^2}{\eta(\beta\phi_\pi - 1)} \left[\left(1 - \frac{\chi}{G\lambda^s}\right) \eta(\pi - 1) + (2\pi - 1) \right. \tag{36}$$
$$\left. -\beta\phi_\pi \left(\frac{\frac{\partial \Pi}{\partial b_t} C(2\pi - 1) - \frac{\partial \mathcal{C}}{\partial b_t}\pi(\pi - 1)}{\frac{\partial \Pi}{\partial b_t} C + \frac{\partial \mathcal{C}}{\partial b_t}\pi} \right) \right].$$

In the case of open-loop monetary strategies, (36) becomes

$$b = -\frac{\gamma\pi^2}{\eta} \left[\left(1 - \frac{\chi}{G\lambda^s}\right) \eta(\pi - 1) + (2\pi - 1) \right]. \tag{37}$$

[14] We substitute the Lagrangian multipliers for the monetary rule, λ^i, by using the first-order condition relative to the nominal interest rate. Even if we require $\phi_\pi > 1/\beta$ across all monetary policy regimes, equation (34) nests the case of open-loop monetary strategies for $\phi_\pi = 0$. Under open-loop strategies, the central bank always adjusts its inflation target in such a way that the nominal interest rate does not respond to the actions of the fiscal authority. Hence, it is as if ϕ_π was zero in (34).

[15] We substitute for λ^p and λ^f by using the first-order conditions relative to the marginal cost and government expenditures.

Table 3: Steady state of the policy game

		Open-loop	Taylor $\phi_\pi = 1.5$ $\pi^\star = 1$	Closed-loop $\phi_\pi = 1.5$
Variable	Symbol	Value		
Consumption	C	0.7486	0.7227	0.7222
Government expenditures	G	0.1366	0.1227	0.1300
Hours worked	N	0.8853	0.8454	0.8522
Debt-to-GDP ratio (annualized)	$b/(4Y)$	-62.13%	112.77%	81.59%
Income tax	τ	0.1423	0.2093	0.2036
Gross inflation	π	0.9973	1	1.0021

The fiscal authority has an incentive to use inflation to achieve two goals: improve the public accounts and bring output close to its efficient level. The equilibrium level of debt is such that, net of the resource cost, the temptation to raise inflation above its steady state to close the output gap is exactly compensated for by the incentive to reduce it to raise real revenues – something that arises only when the government is a creditor. When choosing its policy, the central bank internalizes (37) and trades off the resource cost of inflation against the benefits of affecting b. For example, $\pi = 1$ would imply that $b = -\gamma/\eta = -1.8182$. Instead, $\pi = 0.9973$ and $b = -2.2001$ at the optimal steady state. For mild levels of deflation, the fiscal authority has a greater incentive to rely on inflation to expand output because, as opposed to the case of price stability, a marginal increase in inflation lowers the resource cost. Hence, a higher level of assets is needed for π to be a steady state. The central bank thus accepts some deflation to increase public assets, reduce taxes and improve welfare. When prices are flexible or the output gap is zero ($\eta \to \infty$), the optimal level of debt is zero, as highlighted by Debortoli and Nunes (2012).

In the case of a simple Taylor-type rule with $\pi^\star = \pi = 1$, (36) simplifies to

$$b = \frac{\gamma}{\eta(\beta\phi_\pi - 1)}\left(1 - \beta\phi_\pi \frac{\frac{\partial \Pi}{\partial b_t} C}{\frac{\partial \Pi}{\partial b_t} C + \frac{\partial \mathcal{C}}{\partial b_t}}\right). \tag{38}$$

In contrast to the case of open-loop strategies, the real interest rate rises if inflation deviates from its target $\pi^\star$. The central bank's response generates two counteracting effects. On the one hand, it makes inflationary policies costly by reducing the government's real revenues. Such a cost increases in ϕ_π and b as highlighted in (34). On the other hand, the deterioration in the public accounts triggers debt accumulation, which in turn encourages the future fiscal authority to reduce inflation. Hence, expected inflation falls and the inflation-output trade-off improves. This effect is captured by the term $\partial\Pi/\partial b_t$, which is indeed negative for our calibration. In this respect, a larger ϕ_π

makes inflationary policies less costly. At the optimal steady state, debt is such that the gain from inflating to close the output gap is exactly compensated for by the gain from reducing debt-servicing costs – something that arises only when the government is a debtor. In Figure 1, we show the debt-to-GDP ratio, measured on the vertical axis in percentage points, as a function of the inflation coefficient ϕ_π measured on the horizontal axis. Overall, the cost of implementing inflationary policies increases in ϕ_π. Therefore, the stronger the response of the nominal interest rate, the lower will be the level of debt.

In the case of closed-loop strategies, monetary policy is more flexible: the central bank can vary the interest rate in response to the shocks and the actions of the fiscal authority. This flexibility gives the central bank additional leverage to discourage inflationary fiscal policies. As with Taylor-type rules, the real interest rate increases and the public accounts deteriorate if inflation is above target. In addition, the central bank generates expected inflation when the government accumulates debt. In fact, $\partial\Pi/\partial b_t$ and the numerator on the right-hand side of (35) are positive: the inflation-output trade-off worsens as b increases. This outcome can be achieved by committing to temporarily raising the inflation target in the future. Since inflationary policies both reduce real revenues and worsen the inflation-output trade-off, they are more costly for any given ϕ_π, as compared to Taylor-type rules. Therefore, debt is lower, as is shown in Figure 1. Finally, the central bank deviates from price stability. For example, Table 3 shows that, for $\phi_\pi = 1.5$, a mildly positive inflation rate is optimal. This is because it makes the fiscal authority less willing to rely on inflation, and a lower level of debt is needed to discipline discretionary fiscal behaviour. We find, however, that irrespective of ϕ_π, the inflation rate is small, and we conclude that price stability is roughly optimal.

6.2. Impulse responses

We inspect the dynamics of the model by reporting impulse-response functions of the variables following an i.i.d. technology shock, under the three monetary policy regimes defined in Section 5. Figures 2, 3 and 4 compare the FR equilibrium with the cases of open-loop, closed-loop and simple Taylor-type strategies, respectively. The blue circled lines represent the FR equilibrium; the red starred lines represent the equilibria defined in Section 5. For the closed-loop and the Taylor-type rules, we assume that $\phi_\pi = 1.5$. In each figure, we shock both economies at the steady state and, for the FR case, we choose a steady-state level of debt equal to the steady state of the alternative monetary regime under consideration.[16] We fix the size of the shock to the typical standard deviation considered in the business-cycle literature, 0.0071. Then we normalize consumption, government expenditure, hours worked and output with respect to the shock and report them in percentage deviations from the steady state.

[16] In the FR case, for any chosen steady-state level of debt, there exists an initial condition b_{-1} that supports it. We do not analyze the transition from $t = 0$ to the steady state.

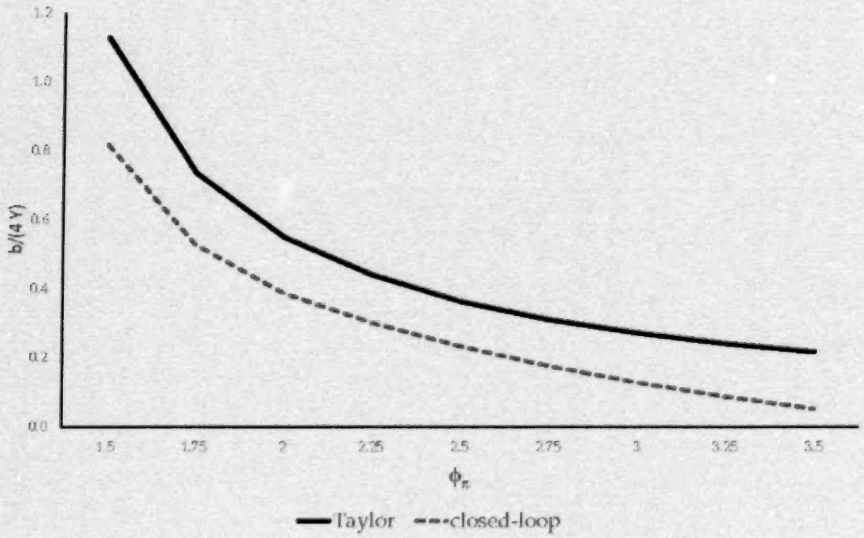

Figure 1: Debt-to-GDP (annualized) ratio as a function of ϕ_π

Hence, a 1% increase in a given variable means that the variable increases as much as productivity. Inflation and the nominal interest rate are not normalized. Rather, they are expressed as deviations from the steady state and in percentage points, so that they can be read as rates. Finally, tax rates and real debt are not normalized either, and they are reported in percentage deviations from their steady-state values. We limit our attention to the case where $\gamma = 20$ and price adjustments are costly.[17]

Figure 2 evaluates the open-loop monetary strategy against the FR model. In the latter regime inflation is stabilized, since changing prices is costly. The real interest rate falls, which leads to lower revenues, because the government is a net creditor ($b < 0$). Labour income taxes remain fairly stable so that hours worked are stabilized, and the nominal interest rate is reduced so as to raise private consumption. Increased tax revenues from higher wages finance an increase in public spending. The response of output and consumption, both private and public, approximate well Pareto efficiency. Inflation stabilization has two consequences. First, since the government is a creditor, it generates a budget deficit in the short run. Second, it induces a unit root in public debt that turns short-run budget imbalances into long-run debt changes. By comparing

[17]The Ramsey problem with flexible prices has been analyzed by Lucas and Stokey (1983) and Chari et al. (1991). If initial nominal public assets are negative, the optimal price level at time zero is infinite so that the distorting labour income tax is reduced. If initial nominal public assets are positive, the optimal monetary policy at time zero is the one that implements the efficient allocation. We do not repeat this analysis here.

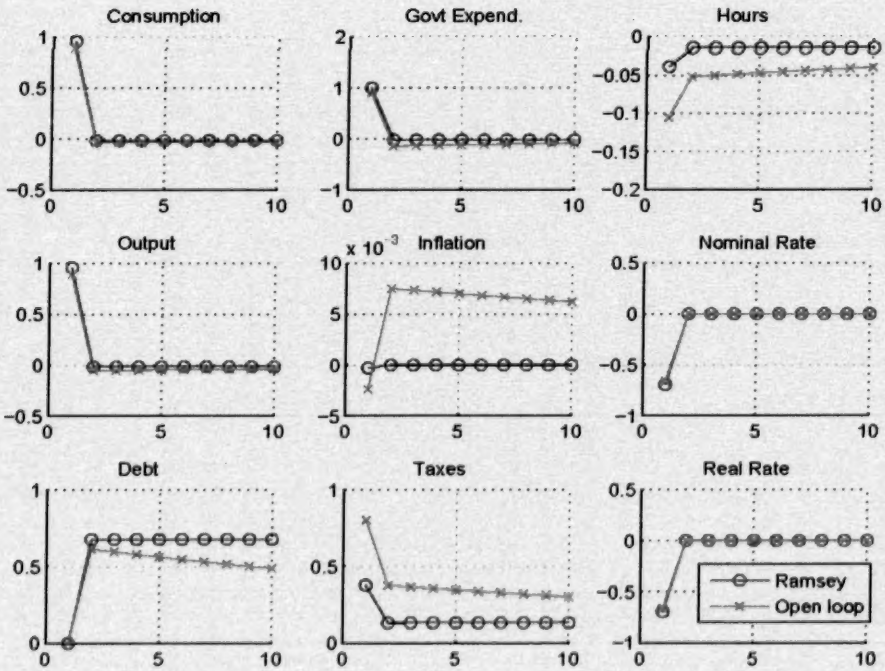

Figure 2: Impulse responses to a technology shock: Full Ramsey and open-loop strategy

the open-loop strategy regime with the FR case, three facts stand out. Fiscal discretion worsens the trade-off between stabilizing inflation and real activity, as becomes clear when we look at the response of hours worked. Should the current fiscal authority limit the tax rise, future governments would be endowed with lower credit, and they would find it optimal to inflate, as we explain in the steady-state section. Since expected inflation worsens the current inflation-output trade-off, the government decides to raise taxes to contain public deficits and future inflation. As a result, hours worked fall. Such a trade-off is absent when a balanced budget is assumed.[18] Second, a lack of fiscal commitment makes inflation more volatile, but the difference is quantitatively negligible: the central bank is not willing to give up on inflation stabilization, despite the additional trade-off that fiscal discretion induces. Finally, government expenditure is barely used for stabilization under either regime, as its responses resemble those under Pareto efficiency. If anything, public spending increases less and debt is overstabilized when the fiscal authority cannot commit to future policies.

[18]See Gnocchi (2013) for the case of monetary commitment and fiscal discretion under a balanced budget.

Figure 3 shows impulse responses under the closed-loop monetary strategy. To begin with, differences across Figures 2 and 3 in the FR stabilization plan are entirely driven by the steady-state level of debt. The government is a net debtor ($b > 0$) and inflation stabilization, which is achieved by lowering the interest rate, generates a budget surplus. Hence, government debt and taxes permanently fall. As in the case with negative debt, inflation is stabilized, and even if inflation volatility becomes greater under fiscal discretion, differences across regimes are quantitatively negligible. Furthermore, the responses of public and private consumption, hours worked, output and debt closely match the ones under FR. In particular, hours worked are fairly stable and the stabilization trade-off is significantly milder than the one arising under open-loop strategies. This is because the monetary authority threatens to raise the nominal interest rate if the fiscal authority deviates from the optimal stabilization plan and pushes inflation above the central bank's target. A higher interest rate would cause a deterioration in the public accounts and would thus be costly for the government. The threat discourages fiscal deviations, and the trade-off induced by the lack of fiscal commitment is weakened. Therefore, even if fiscal deviations are never observed, the threat allows the central bank to sustain an equilibrium that is closer to the FR plan, as compared with the open-loop strategy. Also, debt remains stationary, but its persistency is so high that it is hardly distinguishable from a random walk and its dynamics mimic the one observed under FR.

We conclude with Figure 4, inspecting the dynamics under a simple Taylor-type rule with $\phi_\pi = 1.5$ and a zero inflation target $\pi^* = 1$. In contrast with the regimes previously considered, the interest rate target is constant. It is well known that, for finite values of ϕ_π, the monetary policy stance is too tight, as compared with the FR, even when fiscal policy does not suffer from a lack of commitment problem. Hence, consumption and output do not increase as much as they should, hours worked fall, and inflation is not stabilized. The effects of a suboptimally tight monetary stance on fiscal responses are twofold. On the one hand, debt is less volatile than under FR. Since the fall in nominal and real interest rates is dampened, changes in the service of debt are limited as well. On the other hand, the use of tax rates and government expenditures can be rationalized by the suboptimality of monetary policy. Fiscal policy is discretionary but is still optimally chosen. Therefore, public spending becomes extremely volatile and is used to sustain aggregate demand. Higher taxes not only help to finance government expenditures and contain budget deficits, but they also prevent inflation from falling at the cost of a reduction in hours worked and output. As a result, the effect on output is less than half its FR counterpart.

7. Monetary Institutions and Welfare

In this section, we compare monetary regimes from a welfare standpoint. While fiscal policy is always discretionary, monetary policy is committed to either open-loop strategies, closed-loop strategies or the simple Taylor-type rules described earlier.

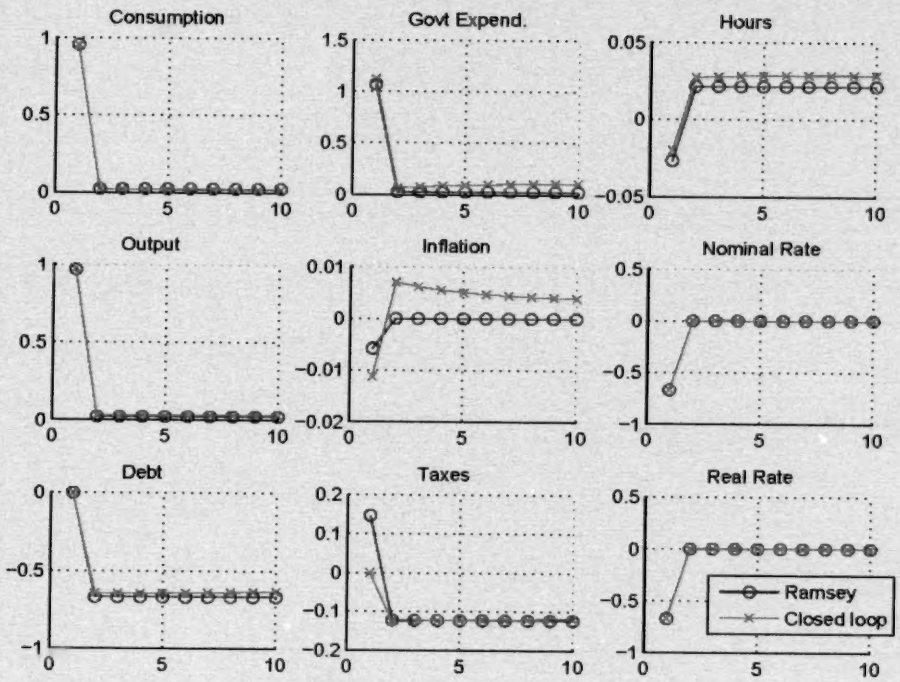

Figure 3: Impulse responses to a technology shock: Full Ramsey and closed-loop strategy, $\phi_\pi = 1.5$

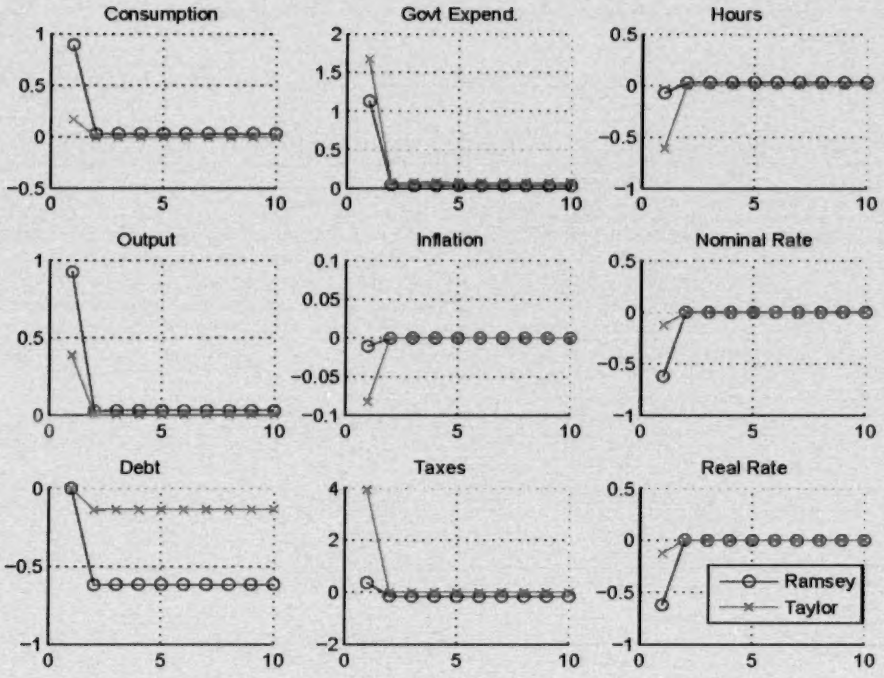

Figure 4: Impulse responses to a technology shock: Full Ramsey and Taylor-type rule, $\phi_\pi = 1.5$

We work with the second-order approximation of the model and focus on welfare, conditional on the deterministic steady state.

Our results can be understood through the lens of our set-up. Because taxes are distortionary, different levels of public debt lead to different allocations and, thus, to different levels of welfare. These differences are of first order and emerge already at the deterministic steady state. Since the three monetary policy regimes have different steady-state levels of public debt, they can be ranked by evaluating welfare at the deterministic steady state. By working with the second-order approximation of the model, we can also evaluate the monetary regimes in terms of their stabilization properties. Because debt plays a key role for welfare and FR does not pin it down uniquely, we evaluate policies relative to the efficient allocation.

Figure 5 plots conditional welfare gains relative to the efficient allocation for the alternative monetary policies. The horizontal axis measures ϕ_π. Welfare is expressed in efficient consumption-equivalent variation – namely, as the percentage of efficient steady-state consumption that the household is willing to give up to be indifferent between the efficient allocation and the monetary regime in question. This percentage

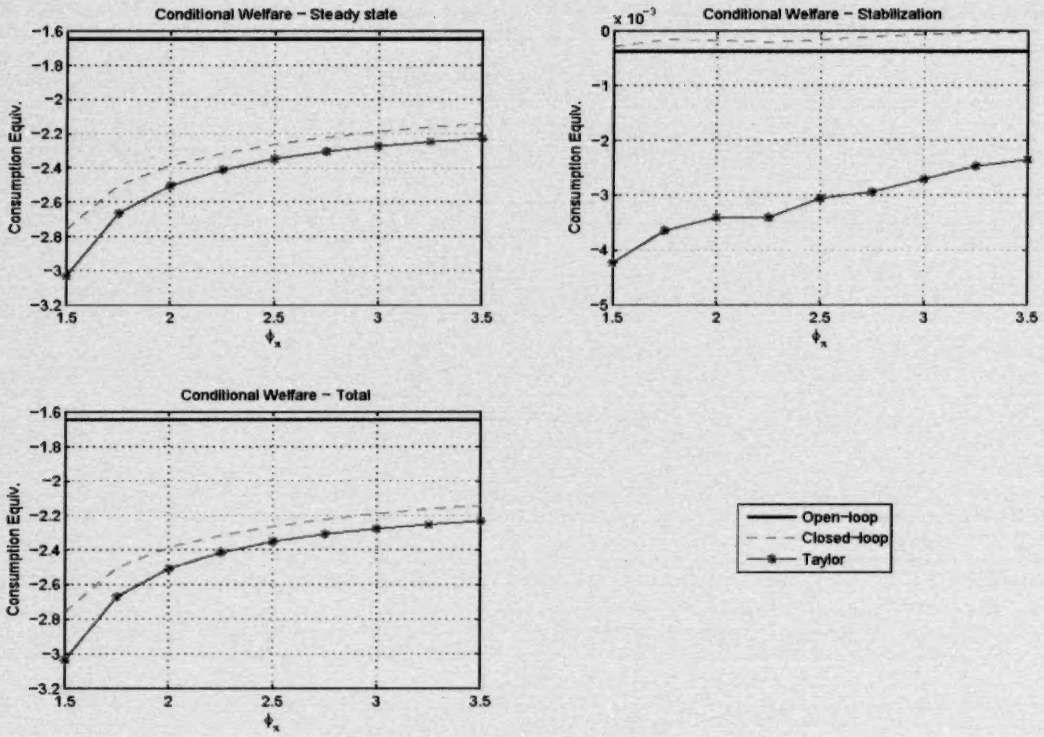

Figure 5: Conditional welfare

is negative in case of welfare losses. The top-left panel of Figure 5 reports the welfare gain at the deterministic steady state, the top-right panel reports the welfare gain from stabilization, and the bottom-left panel reports the total welfare gain, which is the sum of the previous two. Monetary policy under open-loop strategies does not depend on ϕ_π and it dominates both closed-loop and Taylor-type monetary policy. For closed-loop and Taylor rules, total welfare improves as ϕ_π increases. The welfare ranking is driven by the steady-state component, which depends on the level of public debt characterizing each regime. Open-loop strategies achieve the highest welfare because steady-state debt is the lowest. Closed-loop strategies sustain a lower level of debt than the Taylor-type rule and thereby higher welfare. Welfare improves under both regimes as the interest rate response gets higher because this implies lower debt levels. Hence, these welfare results are the mirror image of the steady-state debt results found in Section 6.1.

Closed-loop strategies are characterized by the highest stabilization component, even better than open-loop strategies. As emphasized in Section 6.2, closed-loop strategies sustain equilibrium responses to technology shocks that are much closer to FR than open-loop strategies. This occurs because the government is a debtor ($b > 0$) under

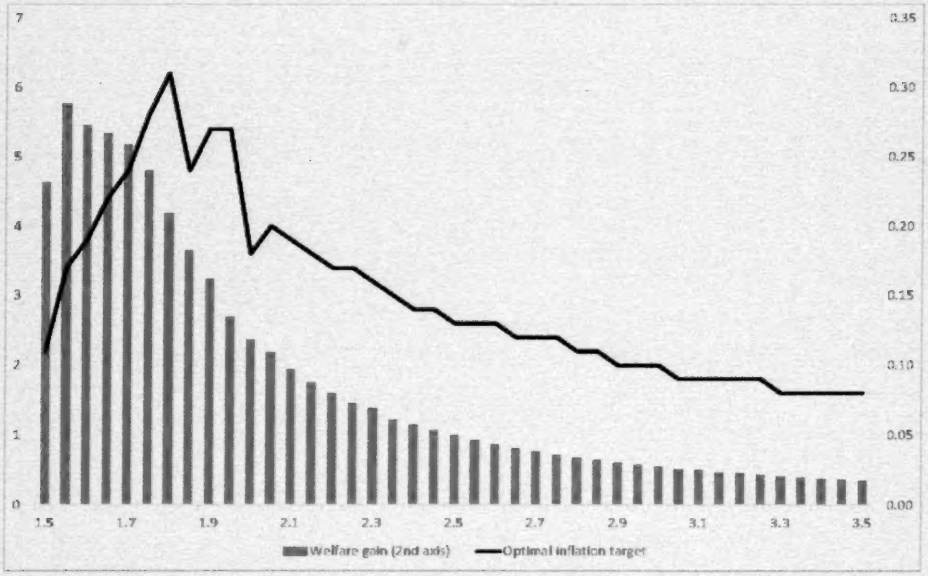

Figure 6: Welfare gains from choosing the inflation target

closed-loop strategies and the monetary authority's threat to raise the nominal interest rate would lead to a deterioration in the government's budget, thereby discouraging fiscal deviations. Taylor ranks worst in terms of stabilization because monetary policy is too tight and fiscal responses strongly suboptimal. We focus on technology shocks, but considering additional exogenous shocks is unlikely to change the ranking in terms of welfare because the stabilization component is small relative to the steady-state counterpart.

What do we learn regarding the design of monetary policy institutions? Two results emerge from our analysis. First, and most important, monetary policy plays a key role in the determination of public debt. Starting from the work of Sargent and Wallace (1981), many contributions have studied how monetary policy affects fiscal policy. Our new insight is that commitment to raising the real interest rate in response to inflation leads to a unique and positive level of debt, and a higher interest rate elasticity to inflation leads to lower debt and higher welfare. Second, the simple interest rate rule studied here approximates relatively well the sophisticated closed-loop rule. Being able to change the state-contingent inflation target and to commit to off-equilibrium responses helps in achieving better stabilization and slightly lower public debt, but it is the interest rate response that pins down debt and welfare.

Optimal monetary policy in the sense of Ramsey is difficult to implement in reality. For this reason, central banks have been adopting simple interest rate rules, which can be easily communicated to and observed by agents. Our Taylor-type rule belongs to this class of monetary policy and it features two parameters: elasticity to the interest rate ϕ_π and the inflation target π^*. Keeping the inflation target constant and equal to

one, an increase in ϕ_π raises welfare, as shown in Figure 5. Changing the elasticity from 1.5 to 3.5 delivers a welfare gain of 0.8 percentage points in consumption-equivalent terms. Changes in the inflation target, however, also affect welfare. An increase in the inflation target is costly because of price-adjustment costs, so that the incentive to raise inflation to close the output gap is reduced. As a result, the steady-state level of debt is lower (in absolute value) and welfare improves.

Figure 6 plots the welfare gain of optimally choosing the inflation target. Net inflation, annual and in percentage points, is measured on the primary vertical axis. The welfare gain is measured as a percentage of efficient steady-state consumption on the secondary vertical axis. With an interest rate elasticity equal to 1.5, setting the net annual inflation target to 2.2 percentage points brings a welfare gain of 0.23% of efficient steady-state consumption relative to keeping the net inflation target equal to zero. For an interest rate elasticity of 1.55, the corresponding figures are 3.4 and 0.29. A couple of comments are in order. First, we report the optimal inflation target for our class of rules: for each value of ϕ_π, we choose the inflation target that maximizes welfare, which we plot as the solid black line.[19] Second, the welfare gains from raising the inflation target first increase and then fall with ϕ_π and are largest at the lower end of the range of interest rate elasticities. The reason is that the high financing costs drive public debt down. Equilibrium public debt is the mirror image of the inflation target: it falls rapidly from 1.8 to 0.4 as the inflation target rises from 2 to 6 percentage points, and then it stabilizes. Given debt, higher inflation is harmful because of price-adjustment costs and higher tax rates due to increased financing costs. The benefits of higher inflation, which stem from its disciplining effects on debt, significantly outweigh the costs only when debt is high – namely, for low values of interest rate elasticity.

8. Conclusions

The design of monetary and fiscal policy institutions emphasizes independence in setting the policy instrument. Central banks are wary to comment on fiscal policy, and governments are not supposed to pressure monetary policy. In the euro area, this independence is reinforced by the prohibition of monetary financing of budgets – namely, the European Central Bank (ECB) is explicitly forbidden to purchase government debt on the primary market. On these grounds, the German Constitutional Court questioned the consistency of the ECB's Outright Monetary Transactions (OMTs) announcement with the European Union primary law. These recent events illustrate the tension behind this division of powers.

Our analysis assumes independence. Nevertheless, we show that monetary and fiscal policy inevitably interact in several dimensions via (i) the strategic set-up; (ii) the ability to commit future actions and responses; and (iii) the general equilibrium effect

[19] We consider $\phi_\pi \in [1.5, 3.5]$ with increments of 0.05. For the inflation target, we consider $\pi^* \in [1, 1.02]$ (at the quarterly frequency) with increments of 0.005.

on the economy. As a result of these interactions, monetary commitment to an interest rate response to deviations of inflation from its target is key for the determination of the level of public debt. An implication of our findings is that the central bank's ability to deliver on its mandate contributes to maintaining low levels of debt and fosters welfare. If limitations placed on the conduct of monetary policy undermine the achievement of the inflation target, fiscal stability might also be compromised. Ultimately, monetary and fiscal policy relate to each other: clearly assigning different instruments to separate authorities is by no means sufficient to strictly separate monetary and fiscal policy. This view has been emphasized by Justice Gerhardt, a member of the German Constitutional Court, who openly disagreed with the majority and questioned the interpretation of OMTs as infringing on the powers of the European member states.

There are several natural extensions to our analysis, which we leave to future research. The effects of monetary policy on the level of debt are of first order, and they affect the steady state. In our analysis, we focus on welfare, conditional on the steady state. However, transitional dynamics are likely to be important. It would be interesting to study whether the transition from a given monetary regime to another one implying a lower level of debt is welfare-improving and thus worth being undertaken. If welfare fell because of the transition, one could analyze whether and which policies could be implemented to make such a transition feasible.

Stabilization issues are also important and the focus of a very large body of literature. We have analyzed the stabilization properties of different monetary policies conditional only on technological shocks. Extending the analysis to additional shocks, such as intertemporal preferences and markup, would be interesting. It could shed light on the ability of each policy to stabilize the economy in response to different shocks. Nevertheless, these effects are bound to remain of second order, and we do not expect them to affect our main first-order results. We used a relatively simple model because the focus of the paper is the strategic interaction of monetary and fiscal policies. We speculate that adding distortions not central to the issue at hand or other state variables, such as capital, will not change the nature of our results.

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Appendix

Appendix A. Full Ramsey

The Lagrangian associated with the Ramsey problem defined in Section 4 is

$$\mathcal{L}^{\mathcal{FR}} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[(1-\chi) \ln C_t + \chi \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \right. \tag{A.1}$$
$$+\lambda_t^f \left[z_t N_t - C_t - G_t - \frac{\gamma}{2}(\pi_t - 1)^2 \right] + \lambda_t^b \left[\frac{1}{C_t(1+i_t)} - \beta \frac{1}{C_{t+1}\pi_{t+1}} \right]$$
$$+\lambda_t^s \left[\frac{b_t}{1+i_t} + \left(mc_t z_t - \frac{N_t^{\varphi} C_t}{1-\chi} \right) N_t - \frac{b_{t-1}}{\pi_t} - G_t \right] + \lambda_t^p \left[\beta \frac{C_t \pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}} \right.$$
$$\left. \left. + \frac{\eta}{\gamma} z_t N_t \left(mc_t - \frac{\eta - 1}{\eta} \right) - \pi_t(\pi_t - 1) \right] \right\}.$$

The first-order conditions relative to variables $C_t, N_t, G_t, \pi_t, mc_t, i_t$ and b_t are

$$\text{FOC } C_t: \quad \frac{1-\chi}{C_t} - \lambda_t^f - \frac{\lambda_t^b}{(1+i_t)C_t^2} + \frac{\lambda_t^p \beta \pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}} - \frac{\lambda_t^s N_t^{1+\varphi}}{1-\chi}$$
$$+\lambda_{t-1}^b \frac{1}{\pi_t C_t^2} - \lambda_{t-1}^p \beta \frac{C_{t-1}\pi_t(\pi_t - 1)}{C_t^2} = 0, \tag{A.2}$$

$$\text{FOC } N_t: \quad -N_t^{\varphi} + z_t \lambda_t^f - \lambda_t^s \left[(1+\varphi) \frac{N_t^{\varphi} C_t}{1-\chi} - mc_t z_t \right] \tag{A.3}$$
$$+\lambda_t^p \frac{\eta}{\gamma} z_t \left(mc_t - \frac{\eta - 1}{\eta} \right) = 0,$$

$$\text{FOC } G_t: \ \frac{\chi}{G_t} - \lambda_t^f - \lambda_t^s = 0, \tag{A.4}$$

$$\text{FOC } \pi_t: \quad -\lambda_t^f \gamma(\pi_t - 1) + \lambda_t^s \frac{b_{t-1}}{\pi_t^2} - \lambda_t^p (2\pi_t - 1) \tag{A.5}$$
$$+\lambda_{t-1}^b \frac{1}{\pi_t^2 C_t} + \lambda_{t-1}^p \frac{C_{t-1}}{C_t}(2\pi_t - 1) = 0,$$

$$\text{FOC } mc_t: \ \lambda_t^s N_t z_t + \lambda_t^p \frac{\eta}{\gamma} z_t N_t = 0, \tag{A.6}$$

$$\text{FOC } i_t: \ -\frac{\lambda_t^b}{C_t(1+i_t)^2} - \frac{\lambda_t^s b_t}{(1+i_t)^2} = 0, \tag{A.7}$$

$$\text{FOC } b_t: \ \frac{\lambda_t^s}{1+i_t} - \frac{\lambda_{t+1}^s \beta}{\pi_{t+1}} = 0. \tag{A.8}$$

Equations (A.2) to (A.7), together with the first-order conditions relative to the Lagrangian multipliers $\lambda_t^f, \lambda_t^b, \lambda_t^s, \lambda_t^p$, form a system of eleven equations in eleven variables.

Appendix B. Fiscal Discretion and Monetary Commitment

Appendix B.1. Open-loop monetary policy strategies

Under open-loop monetary strategies, the fiscal authority can take the interest rate as given. The Lagrangian associated with the fiscal problem is

$$\mathcal{L}^{fO}(b_{t-1}, s^t) = \left[(1-\chi)\ln C_t + \chi \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right] \tag{B.1}$$
$$+\lambda_t^{fO}\left[z_t N_t - C_t - G_t - \frac{\gamma}{2}(\pi_t - 1)^2\right] + \lambda_t^{bO}\left[\frac{1}{C_t(1+i_t)} - \beta E_t \frac{1}{C_{t+1}\pi_{t+1}}\right]$$
$$+\lambda_t^{sO}\left[\frac{b_t}{1+i_t} + \left(mc_t z_t - \frac{N_t^{\varphi} C_t}{1-\chi}\right) N_t - \frac{b_{t-1}}{\pi_t} - G_t\right] + \lambda_t^{pO}\left[\beta E_t \frac{C_t \pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}\right.$$
$$\left.+\frac{\eta}{\gamma} z_t N_t \left(mc_t - \frac{\eta - 1}{\eta}\right) - \pi_t(\pi_t - 1)\right] + \beta E_t \mathcal{L}^{fO}(b_t, s^{t+1}).$$

The first-order conditions of the fiscal policy problem relative to variables C_t, N_t, G_t, π_t, mc_t and b_t are

$$\text{FOC } C_t: \quad \frac{1-\chi}{C_t} - \lambda_t^{fO} - \frac{\lambda_t^{bO}}{C_t^2(1+i_t)} + \lambda_t^{pO} E_t\left[\beta \frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}\right] \tag{B.2}$$
$$-\frac{\lambda_t^{sO} N_t^{1+\varphi}}{1-\chi} = 0,$$

$$\text{FOC } N_t: \quad -N_t^{\varphi} + \lambda_t^{fO} z_t - \lambda_t^{sO}\left[(1+\varphi)\frac{N_t^{\varphi} C_t}{1-\chi} - mc_t z_t\right] \tag{B.3}$$
$$+\lambda_t^{pO}\frac{\eta}{\gamma} z_t \left(mc_t - \frac{\eta - 1}{\eta}\right) = 0,$$

$$\text{FOC } G_t: \ \frac{\chi}{G_t} - \lambda_t^{fO} - \lambda_t^{sO} = 0, \tag{B.4}$$

$$\text{FOC } \pi_t: \quad -\lambda_t^{fO}\gamma(\pi_t - 1) + \lambda_t^{sO}\frac{b_{t-1}}{\pi_t^2} - \lambda_t^{pO}(2\pi_t - 1) = 0, \tag{B.5}$$

$$\text{FOC } mc_t: \ \lambda_t^{pO} z_t N_t + \frac{\lambda_t^{sO}\gamma}{\eta} z_t N_t = 0, \tag{B.6}$$

$$\text{FOC } b_t: \ \beta\lambda_t^{bO} E_t \frac{1}{(C_{t+1}\pi_{t+1})^2}\left[\frac{\partial \mathcal{C}^O}{\partial b_t}\pi_{t+1} + \frac{\partial \Pi^O}{\partial b_t} C_{t+1}\right] + \frac{\lambda_t^{sO}}{1+i_t} - \beta\frac{\lambda_{t+1}^{sO}}{\pi_{t+1}} \tag{B.7}$$
$$+\beta\frac{\lambda_t^{pO} C_t}{C_{t+1}^2} E_t\left[\frac{\partial \Pi^O}{\partial b_t}(2\pi_{t+1} - 1)C_{t+1} - \frac{\partial \mathcal{C}^O}{\partial b_t}\pi_{t+1}(\pi_{t+1} - 1)\right] = 0.$$

Expressions $\partial \mathcal{C}^O/\partial b_t$ and $\partial \Pi^O/\partial b_t$ denote the derivative of functions $C_{t+1}(s^{t+1}, b_t)$ and $\Pi_{t+1}(s^{t+1}, b_t)$, respectively, relative to the level of debt. To simplify the monetary policy problem, we reduce the system (B.2)-(B.7) to four equations by using (B.4) and (B.6) to substitute for λ_t^{fO} and λ_t^{pO}:

$$\frac{1-\chi}{C_t} + \lambda_t^{sO}\left[1 - \frac{N_t^{1+\varphi}}{1-\chi} + \frac{\gamma}{\eta}E_t\beta\frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}\right] - \frac{\lambda_t^{bO}}{C_t^2(1+i_t)} - \frac{\chi}{G_t} = 0, \tag{B.8}$$

$$z_t\frac{\chi}{G_t} - \frac{\lambda_t^{sO}z_t}{\eta} - (1+\varphi)\frac{\lambda_t^{sO}N_t^{\varphi}C_t}{1-\chi} - N_t^{\varphi} = 0, \tag{B.9}$$

$$\lambda_t^{sO}\left[\gamma(\pi_t - 1) + \frac{b_{t-1}}{\pi_t^2} + \frac{\gamma}{\eta}(2\pi_t - 1)\right] - \gamma(\pi_t - 1)\frac{\chi}{G_t} = 0, \tag{B.10}$$

$$\frac{\lambda_t^{sO}}{1+i_t} - \beta\lambda_t^{sO}\frac{\gamma C_t}{\eta}E_t\left[\frac{\partial\Pi^O}{\partial b_t}\frac{2\pi_{t+1}-1}{C_{t+1}} - \frac{\partial\mathcal{C}^O}{\partial b_t}\frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}^2}\right] + \beta\lambda_t^{bO}E_t\left[\frac{\partial\mathcal{C}^O}{\partial b_t}\frac{1}{C_{t+1}^2\pi_{t+1}} + \frac{\partial\Pi^O}{\partial b_t}\frac{1}{C_{t+1}\pi_{t+1}^2}\right] - \beta E_t\frac{\lambda_{t+1}^{sO}}{\pi_{t+1}} = 0. \tag{B.11}$$

Finally, the Lagrangian associated with the monetary policy problem is

$$\begin{aligned}
\mathcal{L}^{\mathcal{MO}} = E_0\Bigg\{\sum_{t=0}^{\infty}\beta^t\Bigg[&(1-\chi)\ln C_t + \chi\ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi}\Bigg] \\
&+\bar{\lambda}_t^{fO}\left[z_tN_t - C_t - G_t - \frac{\gamma}{2}(\pi_t-1)^2\right] + \bar{\lambda}_t^{bO}\left[-\frac{1-\chi}{C_t(1+i_t)} + \beta\frac{1-\chi}{C_{t+1}\pi_{t+1}}\right] \\
&+\lambda_t^{sO}\left[\frac{b_t}{1+i_t} + \left(mc_tz_t - \frac{N_t^{\varphi}C_t}{1-\chi}\right)N_t - \frac{b_{t-1}}{\pi_t} - G_t\right] + \bar{\lambda}_t^{pO}\left[\beta\frac{C_t\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}\right. \\
&\left.+\frac{\eta}{\gamma}z_tN_t\left(mc_t - \frac{\eta-1}{\eta}\right) - \pi_t(\pi_t-1)\right] \\
&+\bar{\lambda}_t^{1O}\left[\frac{1-\chi}{C_t} + \lambda_t^{sO}\left(1 - \frac{N_t^{1+\varphi}}{1-\chi} + \frac{\gamma}{\eta}\beta\frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}\right) - \frac{\lambda_t^{bO}}{C_t^2(1+i_t)} - \frac{\chi}{G_t}\right] \\
&+\bar{\lambda}_t^{2O}\left[z_t\frac{\chi}{G_t} - \frac{\lambda_t^{sO}z_t}{\eta} - (1+\varphi)\frac{\lambda_t^{sO}N_t^{\varphi}C_t}{1-\chi} - N_t^{\varphi}\right] \\
&+\bar{\lambda}_t^{3O}\left[\lambda_t^{sO}\left(\gamma(\pi_t-1) + \frac{b_{t-1}}{\pi_t^2} + \frac{\gamma}{\eta}(2\pi_t-1)\right) - \gamma(\pi_t-1)\frac{\chi}{G_t}\right] \\
&+\bar{\lambda}_t^{4O}\left[\frac{\lambda_t^{sO}}{1+i_t} - \beta\lambda_t^{sO}\frac{\gamma C_t}{\eta}E_t\left(\frac{\partial\Pi^O}{\partial b_t}\frac{2\pi_{t+1}-1}{C_{t+1}} - \frac{\partial\mathcal{C}^O}{\partial b_t}\frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}^2}\right)\right. \\
&\left.+\beta\lambda_t^{bO}E_t\left(\frac{\partial\mathcal{C}^O}{\partial b_t}\frac{1}{C_{t+1}^2\pi_{t+1}} + \frac{\partial\Pi^O}{\partial b_t}\frac{1}{C_{t+1}\pi_{t+1}^2}\right) - \beta E_t\frac{\lambda_{t+1}^{sO}}{\pi_{t+1}}\right]\Bigg\},
\end{aligned} \tag{B.12}$$

where the λs are the Lagrangian multipliers on the constraints in the monetary policy problem and the λs are the Lagrangian multipliers on the constraints in the fiscal policy problem. We compute first-order conditions relative to $C_t, N_t, G_t, \pi_t, mc_t, b_t, i_t$ and λs.

Appendix B.2. Closed-loop monetary policy strategies

Under closed-loop monetary strategies, the nominal interest rate responds to fiscal instruments according to rule (27), which is internalized by the fiscal authority as a constraint. Hence, the Lagrangian associated with the fiscal policy problem becomes

$$\begin{aligned}\mathcal{L}^{fC}(b_{t-1}, s^t) = &\left[(1-\chi)\ln C_t + \chi \ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right] \\ &+\lambda_t^{fC}\left[z_t N_t - C_t - G_t - \frac{\gamma}{2}(\pi_t - 1)^2\right] + \lambda_t^{bC}\left[\frac{1}{C_t(1+i_t)} - \beta E_t \frac{1}{C_{t+1}\pi_{t+1}}\right] \\ &+\lambda_t^{sC}\left[\frac{b_t}{1+i_t} + \left(mc_t z_t - \frac{N_t^{\varphi} C_t}{1-\chi}\right) N_t - \frac{b_{t-1}}{\pi_t} - G_t\right] + \lambda_t^{pC}\left[\beta E_t \frac{C_t \pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}\right. \\ &\left.+\frac{\eta}{\gamma} z_t N_t \left(mc_t - \frac{\eta - 1}{\eta}\right) - \pi_t(\pi_t - 1)\right] \\ &+\lambda_t^{iC}\left[-(1+i_t) + (1+i_t^T)\left(\frac{\pi_t}{\pi_t^T}\right)^{\phi_\pi}\right] + \beta E_t \mathcal{L}^{fC}(b_t, s^{t+1}),\end{aligned} \tag{B.13}$$

where i_t^T, π_t^T and ϕ_π are taken as given by the fiscal authority. Most of the first-order conditions for the fiscal policy problem coincide with the ones associated with Lagrangian (B.1). Nevertheless, for the sake of clarity, we report all of them below. In particular, first-order conditions relative to variables $C_t, N_t, G_t, \pi_t, mc_t, i_t$ and b_t are

$$\begin{aligned}\text{FOC } C_t: \quad &\frac{1-\chi}{C_t} - \lambda_t^{fC} - \frac{\lambda_t^{bC}}{C_t^2(1+i_t)} + \lambda_t^{pC} E_t\left[\beta \frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}\right] \\ &- \frac{\lambda_t^{sC} N_t^{1+\varphi}}{1-\chi} = 0,\end{aligned} \tag{B.14}$$

$$\begin{aligned}\text{FOC } N_t: \quad &-N_t^{\varphi} + \lambda_t^{fC} z_t - \lambda_t^{sC}\left[(1+\varphi)\frac{N_t^{\varphi} C_t}{1-\chi} - mc_t z_t\right] \\ &+\lambda_t^{pC}\frac{\eta}{\gamma} z_t \left(mc_t - \frac{\eta - 1}{\eta}\right) = 0,\end{aligned} \tag{B.15}$$

$$\text{FOC } G_t: \ \frac{\chi}{G_t} - \lambda_t^{fC} - \lambda_t^{sC} = 0, \tag{B.16}$$

$$\begin{aligned}\text{FOC } \pi_t: \quad &-\lambda_t^{fC}\gamma(\pi_t - 1) + \lambda_t^{sC}\frac{b_{t-1}}{\pi_t^2} - \lambda_t^{pC}(2\pi_t - 1) \\ &+\lambda_t^{iC}\frac{\phi_\pi(1+i_t^T)}{\pi_t^T}\left(\frac{\pi_t}{\pi_t^T}\right)^{\phi_\pi - 1} = 0,\end{aligned} \tag{B.17}$$

$$\text{FOC } mc_t\text{: } \lambda_t^{pC} z_t N_t + \frac{\lambda_t^{sC}\gamma}{\eta} z_t N_t = 0, \tag{B.18}$$

$$\text{FOC } i_t\text{: } -\frac{\lambda_t^{bC}}{C_t(1+i_t)^2} - \frac{\lambda_t^{s} b_t}{(1+i_t)^2} - \lambda_t^{iC} = 0, \tag{B.19}$$

$$\begin{aligned}\text{FOC } b_t\text{: } &\beta\lambda_t^{bC} E_t \frac{1}{(C_{t+1}\pi_{t+1})^2}\left[\frac{\partial \mathcal{C}^C}{\partial b_t}\pi_{t+1} + \frac{\partial \Pi^C}{\partial b_t} C_{t+1}\right] + \frac{\lambda_t^{sC}}{1+i_t} - \beta\frac{\lambda_{t+1}^{sC}}{\pi_{t+1}} \\ &+\beta\frac{\lambda_t^{pC} C_t}{C_{t+1}^2} E_t\left[\frac{\partial \Pi^C}{\partial b_t}(2\pi_{t+1}-1)C_{t+1} - \frac{\partial \mathcal{C}^C}{\partial b_t}\pi_{t+1}(\pi_{t+1}-1)\right] = 0.\end{aligned} \tag{B.20}$$

Expressions $\partial \mathcal{C}^C/\partial b_t$ and $\partial \Pi^C/\partial b_t$ denote the derivatives of functions $C_{t+1}(s^{t+1}, b_t)$ and $\Pi_{t+1}(s^{t+1}, b_t)$, respectively, relative to the level of debt. Equations (B.14)-(B.16), (B.18) and (B.20) coincide with their counterparts (B.2)-(B.4), (B.6) and (B.7) for the case of open-loop strategies. Equations (B.17) and (B.5) differ because of the endogenous response of the nominal interest rate. In addition, equation (B.17) coincides with its counterpart for the case of Taylor-type rules discussed below if $\pi_t^T = \pi^*$ and $(1+i_t^T) = \pi^*/\beta$ for all t. To keep the monetary policy problem as tractable as possible, we simplify the system of first-order conditions. First, in equation (B.17), we make use of the relation

$$\frac{\phi_\pi(1+i_t^T)}{\pi_t^T}\left(\frac{\pi_t}{\pi_t^T}\right)^{\phi_\pi - 1} = \frac{\phi_\pi(1+i_t)}{\pi_t},$$

which directly follows from monetary rule (27). Moreover, substituting for λ_t^{iC} from equation (B.19), we obtain

$$-\frac{\lambda_t^{sC}}{\pi_t^2}\left[\frac{\phi_\pi \pi_t b_t}{(1+i_t)} - b_{t-1}\right] - \frac{\phi_\pi \lambda_t^{bC}}{C_t \pi_t (1+i_t)} - \lambda_t^{fC}\gamma(\pi_t - 1) - \lambda_t^{pC}(2\pi_t - 1) = 0. \tag{B.21}$$

We further simplify the system (B.14)-(B.20) by using equations (B.16) and (B.18) to substitute for λ_t^{fC} and λ_t^{pC}:

$$\begin{aligned}\frac{1-\chi}{C_t} &+ \lambda_t^{sC}\left[1 - \frac{N_t^{1+\varphi}}{1-\chi} + \frac{\gamma}{\eta} E_t \beta \frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}\right] \\ &- \frac{\lambda_t^{bC}}{C_t^2(1+i_t)} - \frac{\chi}{G_t} = 0,\end{aligned} \tag{B.22}$$

$$z_t\frac{\chi}{G_t} - \frac{\lambda_t^{sC} z_t}{\eta} - (1+\varphi)\frac{\lambda_t^{sC} N_t^{\varphi} C_t}{1-\chi} - N_t^{\varphi} = 0, \tag{B.23}$$

$$\begin{aligned}&\lambda_t^{sC}\left[\gamma(\pi_t - 1) + \frac{b_{t-1}}{\pi_t^2} - \frac{\phi_\pi b_t}{\pi_t(1+i_t)} + \frac{\gamma}{\eta}(2\pi_t - 1)\right] \\ &-\frac{\phi_\pi \lambda_t^{bC}}{C_t \pi_t (1+i_t)} - \gamma(\pi_t - 1)\frac{\chi}{G_t} = 0,\end{aligned} \tag{B.24}$$

$$\frac{\lambda_t^{sC}}{1+i_t} - \beta\lambda_t^{sC}\frac{\gamma C_t}{\eta}E_t\left[\frac{\partial\Pi^C}{\partial b_t}\frac{2\pi_{t+1}-1}{C_{t+1}} - \frac{\partial\mathcal{C}^C}{\partial b_t}\frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}^2}\right]$$
$$+ \beta\lambda_t^{bC}E_t\left[\frac{\partial\mathcal{C}^C}{\partial b_t}\frac{1}{C_{t+1}^2\pi_{t+1}} + \frac{\partial\Pi^C}{\partial b_t}\frac{1}{C_{t+1}\pi_{t+1}^2}\right] - \beta E_t\frac{\lambda_{t+1}^{sC}}{\pi_{t+1}} = 0. \quad \text{(B.25)}$$

Finally, the Lagrangian associated with the monetary policy problem is

$$\mathcal{L}^{MC} = E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[(1-\chi)\ln C_t + \chi\ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right]\right. \quad \text{(B.26)}$$
$$+\bar{\lambda}_t^{fC}\left[z_tN_t - C_t - G_t - \frac{\gamma}{2}(\pi_t-1)^2\right] + \bar{\lambda}_t^{bC}\left[-\frac{1-\chi}{C_t(1+i_t)} + \beta\frac{1-\chi}{C_{t+1}\pi_{t+1}}\right]$$
$$+\bar{\lambda}_t^{sC}\left[\frac{b_t}{1+i_t} + \left(mc_tz_t - \frac{N_t^{\varphi}C_t}{1-\chi}\right)N_t - \frac{b_{t-1}}{\pi_t} - G_t\right] + \bar{\lambda}_t^{\pi C}\left[\beta\frac{C_t\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}\right.$$
$$\left.+\frac{\eta}{\gamma}z_tN_t\left(mc_t - \frac{\eta-1}{\eta}\right) - \pi_t(\pi_t-1)\right]$$
$$+\bar{\lambda}_t^{1C}\left[\frac{1-\chi}{C_t} + \lambda_t^{sC}\left(1 - \frac{N_t^{1+\varphi}}{1-\chi} + \frac{\gamma}{\eta}\beta\frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}\right) - \frac{\lambda_t^{bC}}{C_t^2(1+i_t)} - \frac{\chi}{G_t}\right]$$
$$+\bar{\lambda}_t^{2C}\left[z_t\frac{\chi}{G_t} - \frac{\lambda_t^{sC}z_t}{\eta} - (1+\varphi)\frac{\lambda_t^{sC}N_t^{\varphi}C_t}{1-\chi} - N_t^{\varphi}\right]$$
$$+\bar{\lambda}_t^{3C}\left[\lambda_t^{sC}\left(\gamma(\pi_t-1) + \frac{b_{t-1}}{\pi_t^2} - \frac{\phi_\pi b_t}{\pi_t(1+i_t)} + \frac{\gamma}{\eta}(2\pi_t-1)\right) - \frac{\phi_\pi\lambda_t^{bC}}{C_t\pi_t(1+i_t)}\right.$$
$$\left.-\gamma(\pi_t-1)\frac{\chi}{G_t}\right]$$
$$+\bar{\lambda}_t^{4C}\left[\frac{\lambda_t^{sC}}{1+i_t} - \beta\lambda_t^{sC}\frac{\gamma C_t}{\eta}E_t\left(\frac{\partial\Pi^C}{\partial b_t}\frac{2\pi_{t+1}-1}{C_{t+1}} - \frac{\partial\mathcal{C}^C}{\partial b_t}\frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}^2}\right)\right.$$
$$\left.\left.+\beta\lambda_t^{bC}E_t\left(\frac{\partial\mathcal{C}^C}{\partial b_t}\frac{1}{C_{t+1}^2\pi_{t+1}} + \frac{\partial\Pi^C}{\partial b_t}\frac{1}{C_{t+1}\pi_{t+1}^2}\right) - \beta E_t\frac{\lambda_{t+1}^{sC}}{\pi_{t+1}}\right]\right\},$$

where the $\bar{\lambda}$s are the Lagrangian multipliers on the constraints in the monetary policy problem, and the λs are the Lagrangian multipliers on the constraints in the fiscal policy problem. We compute first-order conditions relative to $C_t, N_t, G_t, \pi_t, mc_t$, b_t, i_t and λs.

Appendix B.3. Fiscal discretion with Taylor-type rules

$$\mathcal{L}^{fT}(b_{t-1}, s_t) = \left[(1-\chi)\ln C_t + \chi\ln G_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right] \quad \text{(B.27)}$$
$$+\lambda_t^{fT}\left[z_tN_t - C_t - G_t - \frac{\gamma}{2}(\pi_t-1)^2\right] + \lambda_t^{bT}\left[\frac{1}{C_t(1+i_t)} - \beta E_t\frac{1}{C_{t+1}\pi_{t+1}}\right]$$

$$+\lambda_t^{sT}\left[\frac{b_t}{1+i_t}+\left(mc_t z_t-\frac{N_t^{\varphi}C_t}{1-\chi}\right)N_t-\frac{b_{t-1}}{\pi_t}-G_t\right]$$

$$+\lambda_t^{pT}\left[\beta E_t\frac{C_t\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}+\frac{\eta}{\gamma}z_t N_t\left(mc_t-\frac{\eta-1}{\eta}\right)-\pi_t(\pi_t-1)\right]$$

$$+\lambda_t^{iT}\left[-(1+i_t)+\frac{\pi^*}{\beta}\left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi}\right]+\beta E_t\mathcal{L}^{fT}(b_t,s_{t+1}),$$

where λ_t^{iT} is the Lagrangian multiplier on the Taylor-type rule followed by the monetary authority. The fiscal authority maximizes (B.27) relative to $C_t, N_t, G_t, \pi_t,\ mc_t, i_t, b_t$, and the first-order conditions are

$$\text{FOC } C_t\text{:}\ \frac{1-\chi}{C_t}-\lambda_t^{fT}-\frac{\lambda_t^{bT}}{(1+i_t)C_t^2}-\lambda_t^{sT}\frac{N_t^{1+\varphi}}{1-\chi}+\lambda_t^{pT}\beta E_t\frac{\pi_{t+1}(\pi_{t+1}-1)}{C_{t+1}}=0, \quad \text{(B.28)}$$

$$\text{FOC } N_t\text{:}\ -N_t^{\varphi}+\lambda_t^{fT}z_t+\lambda_t^{s}\varphi\left[mc_t z_t-(1+\varphi)\frac{C_t N_t^{\varphi}}{1-\chi}\right]+\lambda_t^{pT}\frac{\eta}{\gamma}z_t\left(mc_t-\frac{\eta-1}{\eta}\right)=0, \quad \text{(B.29)}$$

$$\text{FOC } G_t\text{:}\ \frac{\chi}{G_t}-\lambda_t^{fT}-\lambda_t^{sT}=0, \quad \text{(B.30)}$$

$$\text{FOC } \pi_t\text{:}\ -\lambda_t^{fT}\gamma(\pi_t-1)+\lambda_t^{sT}\frac{b_{t-1}}{\pi_t^2}-\lambda_t^{pT}(2\pi_t-1)+\lambda_t^{iT}\frac{\phi_\pi}{\beta}\left(\frac{\pi_t}{\pi^*}\right)^{\phi_\pi-1}=0, \quad \text{(B.31)}$$

$$\text{FOC } mc_t\text{:}\ \lambda_t^{sT}+\lambda_t^{pT}\frac{\eta}{\gamma}=0, \quad \text{(B.32)}$$

$$\text{FOC } i_t\text{:}\ -\frac{\lambda_t^{bT}}{C_t(1+i_t)^2}-\lambda_t^{sT}\frac{b_t}{(1+i_t)^2}-\lambda_t^{iT}=0, \quad \text{(B.33)}$$

$$\text{FOC } b_t\text{:}\ \beta\lambda_t^{bT}E_t\frac{1}{(C_{t+1}\pi_{t+1})^2}\left[\frac{\partial\mathcal{C}^T}{\partial b_t}\pi_{t+1}+\frac{\partial\Pi^T}{\partial b_t}C_{t+1}\right]+\frac{\lambda_t^{sT}}{1+i_t}-\beta E_t\frac{\lambda_{t+1}^{sT}}{\pi_{t+1}} \quad \text{(B.34)}$$

$$+\beta\lambda_t^{pT}C_t E_t\frac{1}{C_{t+1}^2}\left[C_{t+1}(2\pi_{t+1}-1)\frac{\partial\Pi^T}{\partial b_t}-\pi_{t+1}(\pi_{t+1}-1)\frac{\partial\mathcal{C}^T}{\partial b_t}\right]=0.$$

Expressions $\partial\mathcal{C}^T/\partial b_t$ and $\partial\Pi^T/\partial b_t$ denote the derivatives of functions $C_{t+1}(s^{t+1},b_t)$ and $\Pi_{t+1}(s^{t+1},b_t)$, respectively, relative to the level of debt.

Appendix B.4. Implementation

Let $\bar{\mathcal{A}}$ be a bounded competitive equilibrium. Assume that there exists a unique solution to system (7)-(9), (15), (19)-(20), (26) and (27). If $\{i_t^T,\pi_t^T\}\in\mathcal{A}$ and $\{G_t,\tau_t\}\in\mathcal{A}_t$, the solution to system (7)-(9), (15), (19)-(20), (26) and (27) is $\bar{\mathcal{A}}$. This statement allows us to break the policy problem into two steps, as commonly done in the primal approach: (i) the determination of the optimal allocation; and (ii) the determination of policy rules that implement the desired allocation. The proof of the statement is

trivial. According to the definition of competitive equilibrium, $\bar{\mathcal{A}}$ satisfies (7)-(9), (15), and (19)-(20). By substituting targets and equilibrium inflation in the monetary rule, it can be easily seen that (27) is satisfied as well, while (26) holds by construction. Hence, $\bar{\mathcal{A}}$ is a solution to system (7)-(9), (15), (19)-(20), (26) and (27). If the system admits a unique solution, rules (26) and (27) uniquely implement $\bar{\mathcal{A}}$. Therefore, to guarantee implementation we need to guarantee uniqueness. To this purpose, we check numerically that the Blanchard and Kahn (1980) condition holds and the solution to the system coincides with $\bar{\mathcal{A}}$.

Appendix C. Solution method

We solve the model by resorting to a second-order perturbation method following Schmitt-Grohé and Uribe (2004b), from whom we borrow some notation. Steinsson (2003) first uses a perturbation method to solve for a Markov-perfect equilibrium in a New Kenynesian model. Since his system of equations evaluated at the steady state is independent of the unknown derivatives of decision rules, the approximation order is inconsequential for the accuracy of his steady-state results. Klein et al. (2008) first proposed a perturbation approach for cases similar to ours, where the non-stochastic steady state and the dynamics of the model have to be solved jointly. In these cases, the approximation error of decision rules also affects steady-state results. In principle, one could progressively raise the approximation order until changes in the steady state are small enough, as suggested by Klein et al. (2008). In practice, when the state space is large, high-order approximations are computationally very expensive. Azzimonti et al. (2009) document that in an environment similar to ours a second-order perturbation method is roughly as accurate as a global method. Global methods still perform relatively better to study the transitional dynamics, from which we abstract in this paper.

Appendix C.1. Notation

We first cast the system of equilibrium conditions in the form

$$E_t\{f(x_2', x_2, x_1', x_1, \theta_1)\} = 0, \tag{C.1}$$

where x_1 is an $n_1 \times 1$ vector of predetermined variables at time t and x_2 is an $n_2 \times 1$ vector of non-predetermined variables at time t; $f \equiv [f^1; ...; f^n]$ is an $n \times 1$ vector of functions mapping $\Re^{n_2} \times \Re^{n_2} \times \Re^{n_1} \times \Re^{n_1}$ into $\Re$, $n = n_1 + n_2$; θ_1 is a vector of parameters; primes indicate variables dated in period $t+1$. Vector x_1 is partitioned as $x_1 = [x_{11}; x_{12}]^\top$, where x_{11} collects endogenous variables and x_{12} collects exogenous variables. We assume that x_{12} follows the exogenous stochastic process

$$x_{12}' = \Lambda x_{12} + \tilde{\eta}\sigma\epsilon. \tag{C.2}$$

Both vectors x_{12} and innovation ϵ have size $n_\epsilon \times 1$, and the innovation is independently and identically distributed over time with mean zero and variance-covariance matrix I.

Both $\bar{\eta}$ and Λ are matrices of order $n_\epsilon \times n_\epsilon$ and σ is a scalar. All eigenvalues of matrix Λ are assumed to have modulus less than one. f is defined so that (C.2) holds and its parameters are included in θ_1. The candidate solution to model (C.1) has the form

$$\begin{aligned} x_2 &= G(x_1, \sigma), \\ x_1 &= H(x_1, \sigma) + \eta\sigma\epsilon, \end{aligned} \tag{C.3}$$

where η is a matrix defined as $\eta \equiv [\emptyset; \tilde{\eta}]$ with size $n_1 \times n_\varepsilon$; $G \equiv [G^1; ...; G^{n_2}]$ and $H \equiv [H^1; ...; H^{n_1}]$ are vectors of unknown functions that map $\Re^{n_1}$ into $\Re$. The unknown functions are a solution to (C.1) if they satisfy the functional equation

$$F(x_1, \sigma) \equiv E_t\{f(G(H(x_1, \sigma) + \eta\sigma\epsilon), G(x_1, \sigma), H(x_1, \sigma) + \eta\sigma\epsilon, x)\} = 0 \tag{C.4}$$

for any x_1, where (C.4) obtains from substituting the candidate solution (C.3) into (C.1). The non-stochastic steady state is defined as a couple of vectors $(\bar{x}_1, \bar{x}_2)$ such that $f(\bar{x}_2, \bar{x}_2, \bar{x}_1, \bar{x}_1; \theta_1) = 0$. Notice that $\bar{x}_2 = G(\bar{x}_1, 0)$ and $\bar{x}_1 = H(\bar{x}_1, 0)$ because $\sigma = 0$ and (C.1) imply $E_t f = f = 0$.

Appendix C.2. A classic perturbation approach

Assume that f is continuous and twice continuously differentiable. Then, a second-order Taylor expansion to functions G and H around the non-stochastic steady state can be written as[20]

$$G^i \approx g_0^i + (x_1 - \bar{x}_1)^\top g_1^i + \frac{1}{2}(x_1 - \bar{x}_1)^\top g_2^i (x_1 - \bar{x}_1), \qquad i = 1, ..., n_2; \tag{C.5}$$

$$H^j \approx h_0^j + (x_1 - \bar{x}_1)^\top h_1^j + \frac{1}{2}(x_1 - \bar{x}_1)^\top h_2^j (x_1 - \bar{x}_1) + \eta^j \sigma\epsilon, \qquad j = 1, ..., n_1,$$

where g_0^i and h_0^j are scalars; g_1^i and h_1^j are vectors of order $n_1 \times 1$; g_2^i and h_2^j are matrices of order $n_1 \times n_1$; η^j denotes the j-th row of matrix η. Coefficients in (C.5) relate to functions G^i and H^j and, at this stage, they are unknown. For example, elements of g_1^i are

$$g_1^i(j, 1) = \left.\frac{\partial G^i(x_1, \sigma)}{\partial x_1(j, 1)}\right|_{x_1 = \bar{x}_1, \sigma = 0}. \tag{C.6}$$

In (C.5), there are $n(1 + n_1 + n_1^2)$ unknown coefficients that we stack in a vector denoted by θ_2. Equation (C.4) implies that for all x_1 and σ

$$\left. F_{x_1^k, \sigma^j}(x_1, \sigma)\right|_{x_1 = \bar{x}_1, \sigma = 0} = 0 \qquad \forall j, k \in \{0, 1, 2\}, \tag{C.7}$$

[20] Schmitt-Grohé and Uribe (2004b) prove that the coefficients of cross terms between x_1 and σ are equal to 0. All other non-zero terms that do not explicitly appear in (C.5), such as the second-order terms in σ, are included in g_0^i and h_0^j. For further details we refer to Schmitt-Grohé and Uribe (2004b).

where the left-hand side in (C.7) stands for the derivative of F with respect to x_1 taken k times and with respect to σ taken j times. Schmitt-Grohé and Uribe (2004b) show that equations (C.7) form a system of $n(1+n_1+n_1^2)$ equations that allow us to solve for θ_2 as a function of the non-stochastic steady state and parameter θ_1, i.e.,

$$S(\theta_2, \bar{x}_1, \bar{x}_2, \theta_1) \equiv F_{x_1^k, \sigma^j}(x_1, 0)\Big|_{x_1=\bar{x}_1, \sigma=0} = 0. \tag{C.8}$$

The method boils down to two simple steps:

1. Solve $f(\bar{x}_2, \bar{x}_2, \bar{x}_1, \bar{x}_1, \theta_1) = 0$ to find $\bar{x}_1(\theta_1)$ and $\bar{x}_2(\theta_1)$;
2. Solve $S(\theta_2, \bar{x}_1, \bar{x}_2, \theta_1) = 0$ to find $\theta_2(\bar{x}_1, \bar{x}_2, \theta_1)$.

The two steps can be performed sequentially because the first one does not require knowledge of θ_2.

Appendix C.3. A generalized perturbation approach

Markov-perfect equilibria often present an additional hurdle: the set of conditions f involves the derivatives of unknown functions G^i and H^j. For instance, our system of equations contains $\partial C/\partial b_t$. Following the notation outlined in the previous section, system f includes

$$\frac{\partial G^i(x_1, \sigma)}{\partial x_1}; \quad \frac{\partial H^j(x_1, \sigma)}{\partial x_1}, \tag{C.9}$$

and model (C.1) needs to be redefined as

$$\begin{aligned} &E_t\left\{ f\left(x_2', x_2, x_1', x_1, \frac{\partial G^1(x_1,\sigma)}{\partial x_1}, ..., \frac{\partial G^{n_2}(x_1,\sigma)}{\partial x_1}, \right.\right. \\ &\left.\left. \frac{\partial H^j(x_1,\sigma)}{\partial x_1}, ..., \frac{\partial H^{n_1}(x_1,\sigma)}{\partial x_1}, \theta_1 \right) \right\} = 0, \\ &i = 1, ..., n_2 \quad j = 1, ..., n_1. \end{aligned} \tag{C.10}$$

One can similarly redefine solutions G^i and H^j by rewriting (C.4) accordingly. As in Steinsson (2003) and Klein et al. (2008), the perturbation approach can be naturally extended to accommodate this case. If (C.5) approximates (C.3), then derivatives of G^i and H^j with respect to the state vector x_1 can be approximated as

$$\begin{aligned} \frac{\partial G^i(x_1,\sigma)}{\partial x_1} &\approx g_1^i + g_2^i(x_1 - \bar{x}_1), \\ \frac{\partial H^j(x_1,\sigma)}{\partial x_1} &\approx h_1^j + h_2^j(x_1 - \bar{x}_1). \end{aligned} \tag{C.11}$$

Recall that unknown coefficients θ_2 are found by using (C.6)-(C.8), which involve first- and second-order derivatives of f evaluated at the steady state. If f also contains

functions (C.9), their first- and second-order derivatives are needed. Again applying approximation (C.11) immediately implies that

$$\left.\frac{\partial^2 G^i(x_1,\sigma)}{\partial^2 x_1}\right|_{x_1=\bar{x}_1,\sigma=0} \approx g_2^i, \quad \left.\frac{\partial^2 H^j(x_1,\sigma)}{\partial^2 x_1}\right|_{x_1=\bar{x}_1,\sigma=0} \approx h_2^j, \tag{C.12}$$
$$\left.\frac{\partial^3 G^i(x_1,\sigma)}{\partial^3 x_1}\right|_{x_1=\bar{x}_1,\sigma=0} \approx 0, \quad \left.\frac{\partial^3 H^j(x_1,\sigma)}{\partial^3 x_1}\right|_{x_1=\bar{x}_1,\sigma=0} \approx 0.$$

Since the third derivative is zero for second-order accurate solutions, equation (C.8) contains only the unknown coefficients θ_2, as in the classic perturbation approach. Therefore, *conditional* on knowing $\bar{x}_1$ and $\bar{x}_2$, the second step continues to apply and results by Schmitt-Grohé and Uribe (2004b) still hold. We can then recover $\theta_2(\bar{x}_1, \bar{x}_2, \theta_1)$ by solving

$$S(\theta_2, \bar{x}_1, \bar{x}_2, \theta_1) = 0. \tag{C.13}$$

However, after substituting (C.11) into (C.10), the first step is different because

$$f(\bar{x}_2, \bar{x}_2, \bar{x}_1, \bar{x}_1, \theta_1, \theta_2) = 0 \tag{C.14}$$

only defines implicit functions $\bar{x}_1(\theta_1, \theta_2)$ and $\bar{x}_2(\theta_1, \theta_2)$. Since the non-stochastic steady state now depends on θ_2, the perturbation steps cannot be performed sequentially. We thus need to solve the system (C.13)-(C.14) jointly to find θ_2, $\bar{x}_1$ and $\bar{x}_2$.

Appendix C.4. Numerical algorithm

We expand on the set of routines made publicly available by Schmitt-Grohé and Uribe at `http://www.columbia.edu/~mu2166/2nd_order.htm`. Among others, they provide routines gx_hx.m, gxx_hxx.m and gss_hss.m to solve for coefficients θ_2, given θ_1 and the non-stochastic steady state. We define the following steps:

1. Assume that θ_2 is known;
2. Replace derivatives of G and H with their approximation in f according to (C.11);
3. Compute $\bar{x}_1$ and $\bar{x}_2$ with a non-linear solver using θ_1 and θ_2 as inputs;
4. Use $\bar{x}_1$ and $\bar{x}_2$ as inputs in gx_hx.m, gxx_hxx.m and gss_hss.m to compute θ_2^*;
5. Require consistency by imposing $\theta_2 = \theta_2^*$.

We implement the algorithm in two alternative ways. We perform steps 1 to 5 simultaneously by using a non-linear solver on θ_2, $\bar{x}_1$ and $\bar{x}_2$. Alternatively, we also start with an initial guess on θ_2 and iterate until convergence. When the number of state variables becomes large, as in the case of closed-loop strategies, we fail to reach convergence for some parameter values. In that case, we rely only on the first procedure.